10

Liquids and gases are collectively known as fluids. Fluid flows under the action of an applied force and does not have a shape of its own. It takes the shape of the vessel in which it is placed.

In this chapter, we shall study hydrostatics and hydrodynamics. The study of fluids at rest is known as hydrostatics or **fluid statics.** The study of fluids in motion is named as **hydrodynamics** or **fluid dynamics**.

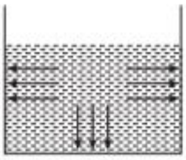
MECHANICAL PROPERTIES OF FLUIDS

|TOPIC 1|
Hydrostatics

The key property of fluids is that they offer very little resistance to shear stress; their shape changes by application of very small shear stress.

THRUST

The molecules of a fluid kept in a container are in random motion due to their thermal velocities. So, they constantly collide with the walls of the container and rebounding from them, suffering a change in momentum normal to the colliding walls due to which some normal change in momentum is transferred to the walls.



Liquid in a container is exerting thrust

The normal change in momentum transferred to the walls of the container per unit time by the molecules of the fluid is called thrust of the fluid on the container.

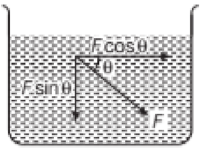
CHAPTER CHECKLIST

- Thrust
- Pressure
- Pascal's Law
- Buoyancy
- Archimedes' Principle
- Surface Tension
- Surface Energy
- Angle of Contact Capillarity
- Bernoulli's Theorem
- Viscosity
- Critical Velocity
- Stoke's Law

Liquid in Equilibrium

When a liquid is in equilibrium, then the net force acting on the liquid always acts perpendicular to its surface.
It can be easily proved as below

Consider a liquid kept in a vessel. Let us suppose that the liquid is in the equilibrium of rest and the net force F makes an angle θ with the direction parallel to the surface of liquid as shown in the figure.



Parallel and perpendicular components of the force F

The force F can be resolved into two components such as

(i) $F\cos\theta$ as tangential component acts parallel to the surface of liquid.

(ii) $F\sin\theta$ as normal component acts perpendicular to the surface of liquid.

Since, the liquid is at rest (i.e. there is no flow of liquid), the tangential component acts parallel to the surface of the liquid must be zero.

i.e. $F\cos\theta = 0$

As F cannot be zero, we have $\cos\theta = 0 \Rightarrow \theta = 90^\circ$

Hence, if a liquid is in equilibrium state of rest, then the net force acting on it is always perpendicular to its surface.

PRESSURE

The pressure of liquid at a point is the thrust (or normal force) exerted by the liquid at rest per unit area around that point. If the total force F acts normally over a flat area A, then, the pressure is

$$p = \frac{\text{thrust}}{\text{area}} \quad \text{or} \quad \boxed{\text{Pressure, } p = \frac{F}{A}}$$

The unit of pressure is **dyne/cm^2** in CGS system and **N/m^2** in SI system and dimensional formula of pressure is $[ML^{-1}T^{-2}]$.

Pressure of 1 N/m^2 is called **1 pascal** in the honour of the French scientist **Blaise Pascal** (1623-1662) who carried out the pioneering studies on fluid pressure.

$$1\,\text{Pa}\ (\text{or } 1\,\text{N/m}^2) = 10\,\text{dyne/cm}^2$$

The another common unit of pressure is atmosphere (atm). 1 atm is defined as the pressure exerted by the atmosphere at sea level.

$$1\ \text{atm} = 1.013 \times 10^5\ \text{Pa}.$$

If the force is not distributed uniformly over the given surface, then pressure will be different at different points. If a force ΔF acts normally on a small area ΔA surrounding a given point, then pressure at that point will be

$$p = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} = \frac{dF}{dA}$$

Pressure is a scalar quantity, because liquid pressure at a particular point in liquid has same magnitude in all directions. This shows that a definite direction is not associated with liquid pressure.

EXAMPLE |1| Pressure Exerted by Human Body

The two thigh bones (femurs), each of cross-sectional area 10 cm^2 support the upper part of a human body of mass 40 kg. Estimate the average pressure sustained by the femurs. **[NCERT]**

Sol. Given, $A = 20 \times 10^{-4}\,\text{m}^2$

Weight of body acting vertically downwards

Force on bones, $F = 40$ kg - wt $= 400$ N $\quad [\because g = 10\ \text{m/s}^2]$

$$p_{av} = \frac{F}{A} = \frac{400}{20 \times 10^{-4}}$$
$$= 2 \times 10^5\ \text{N/m}^2$$

EXAMPLE |2| Fluid Pressure Measurement

Consider a spring having spring constant of 81.75 N/m of pressure measuring device. A force acting on the piston compresses the spring downward and the area of the piston is 3m^2. Find out the compression in the spring, if the upthrust force exerted by the fluid on the piston is 12 N.

Sol. As the force acting on the piston compresses the spring. This force will be balanced by the force acting upward by the fluid known as **upthrust force**.

$\therefore$ $$p = \frac{F}{A} = \frac{kx}{A}$$

Given, $F = 12$ N, $A = 3\text{m}^2$ and $k = 81.75$ N/m

From $F = kx$

$\Rightarrow$ $$x = \frac{F}{k} = \frac{12}{81.75}$$
$$x = 0.146\ \text{m}$$

Hence, compression in the spring is $x = 14.6$ cm

Practical Applications of Pressure

(i) A sharp knife cuts better than a blunt one.

(ii) A camel walks easily on sand while it is difficult to walk on a sand for man.

(iii) Railway tracks are laid on large sized wooden or iron sleepers.

(iv) A sharp needle peers the skin easily but not a dull needle although the force applied in both the cases is the same.

DENSITY AND RELATIVE DENSITY

Density of a substance is defined as mass per unit volume of the substance. It is denoted by ρ.

If M be the mass of a substance of volume V, then density of that substance is given by

$$\text{Density, } \rho = \frac{M}{V}$$

The SI unit of density is kg/m^3 and dimensional formula is $[ML^{-3}]$.

Relative density (or specific gravity) of substance is defined as the ratio of the density of that substance to the density of water at 4°C, i.e.

$$\text{Relative density} = \frac{\text{Density of substance}}{\text{Density of water at } 4°C}$$

It has no unit and no dimensions. It is a positive **scalar quantity**.

The density of water at 4°C is maximum and equal to 1000 kg/m^3.

PASCAL'S LAW

The French scientist **Blaise Pascal** observed that *pressure in* a fluid at rest is the same at all points if they are at same height.

This is known as **Pascal's law**.

Proof of Pascal's Law

It can be proved by using two principles as given below:

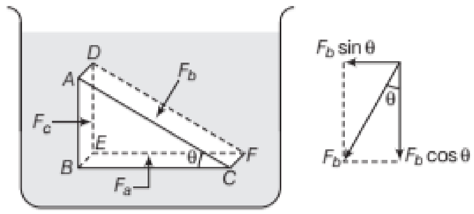
(i) The force on any layer of a fluid at rest is normal to the layer.

(ii) Newton's first law of motion

In given figure, consider an element in the interior of a fluid at rest. This element *ABC-DEF* is in the form of a right angled prism. This prismatic element is very small so that every part of it can be considered at the same depth from the liquid surface and therefore, the effect of the gravity is the same at all these points.

Let we extend this element for our clarification. The forces on this element are those exerted by the rest of the fluid and they must be normal to the surfaces of the element as discussed earlier. Suppose, the fluid exert pressures p_a, p_b and p_c on the faces *BEFC*, *ADFC* and *ADEB*, respectively, of this element and the corresponding normal forces on these faces are F_a, F_b and F_c.

Let A_a, A_b and A_c be the respective areas of the three faces.



Proof of Pascal's law

In right triangle ABC, $\angle ACB = \theta$

Along horizontal direction, $F_b \sin\theta = F_c$

Along vertical direction, $F_b \cos\theta = F_a$ ...(i)

From the geometry, we have

$A_b \sin\theta = A_c$ and $A_b \cos\theta = A_a$ ...(ii)

From the above equations, we have

$$\frac{F_b \sin\theta}{A_b \sin\theta} = \frac{F_c}{A_c} \text{ and } \frac{F_b \cos\theta}{A_b \cos\theta} = \frac{F_a}{A_a}$$

$$\therefore \quad \frac{F_a}{A_a} = \frac{F_b}{A_b} = \frac{F_c}{A_c}$$

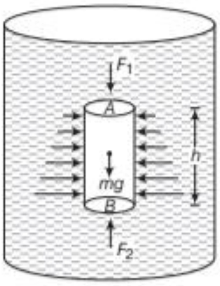
$$\Rightarrow \quad p_a = p_b = p_c$$

Hence, the pressure exerted by the fluid at rest on a body in the fluid is same in all directions. Thus, pressure is not a vector quantity because no direction can be assigned to it.

e.g. Consider a horizontal bar of uniform cross-section is placed in the fluid at rest. The bar is in equilibrium because the horizontal forces exerted at its two ends are balanced or the pressures at the two ends are equal. This proves that for a liquid in equilibrium, the pressure is same at all points in a horizontal plane.

Variation of Pressure with Depth

Consider a fluid at rest having density ρ contained in a cylindrical vessel as shown in figure. Let the two points A and B separated by a vertical distance h.



Fluid under gravity

Now, we consider an imaginary cylinder of fluid of cross-sectional area a, such that points A and B lie on its upper and lower circular faces, respectively. Then, weight of fluid cylinder acting downwards,

$$w = m \times g = \text{volume} \times \text{density} \times g = Ah\rho g$$

where, $V = A \times h$

As the fluid is at rest, the resultant horizontal forces should be zero and the resultant vertical forces should balance the weight of the element.

∴ Net downward force = net upward force

$$F_1 + w = F_2$$

where, F_1 is the force acting downwards at the top and F_2 is the force acting upward at the bottom.

$$F_2 - F_1 = w$$

$$p_2 A - p_1 A = Ah\rho g$$

$$A(p_2 - p_1) = A(h\rho g)$$

$$\boxed{\text{Pressure difference, } p_2 - p_1 = h\rho g}$$

Hence, the pressure difference depends on the vertical height (i.e. distance between point A and point B), density of the fluid and the acceleration due to gravity.

If point A is shifted to the fluid surface, which is open to the atmosphere, then replace p_1 by atmospheric pressure p_a and p_2 by p, we get

$$\boxed{\text{Pressure, } p = p_a + h\rho g}$$

Thus, the pressure p at depth below the surface of a liquid open to the atmosphere is greater than atmospheric pressure by an amount $\rho g h$.

This excess of pressure at depth h in liquid $p - p_a = h\rho g$, i.e. called **gauge pressure** at point B, while point A is at the liquid surface.

Thus, gauge pressure at a point in a fluid is the difference of total pressure at that point and atmospheric pressure.

EXAMPLE |3| A Swimmer Experiences the Pressure

What is the pressure on a swimmer 10 m below the surface of a lake? **[NCERT]**

Sol. Here, $h = 10\,\text{m}$ and $\rho = 1000\,\text{kg/m}^3$

$$g = 10\,\text{m/s}^2$$

$$p = p_a + \rho g h$$

$$= 1.01 \times 10^5 \text{ Pa} + 1000 \text{ kg/m}^3 \times 10 \text{ m/s}^2 \times 10 \text{ m}$$

$$= 2.01 \times 10^5 \text{ Pa} \simeq 2\text{atm}$$

EXAMPLE |4| Pressure at a depth in ocean

At a depth 1000 m in an ocean

(i) What is the absolute pressure?

(ii) What is the gauge pressure?

(iii) Find the force acting on the window of area 20 cm × 20 cm of a submarine at this depth, the interior of which is maintained at sea-level atmospheric pressure. (The density of sea water is $1.03 \times 10^3 \text{kgm}^{-3}$, $g = 10 \text{ ms}^{-2}$) **[NCERT]**

Sol. Given, $h = 1000$ m and $\rho = 1.03 \times 10^3 \text{ kg/m}^3$

We know, $p = p_a + \rho g h$

$$p = 1.01 \times 10^5 + 1.03 \times 10^3 \times 10 \times 1000$$

$$p = 104.01 \times 10^5 \text{ Pa} = 104 \text{ atm}$$

Now, $p_g = p - p_a$

$$p_g = 104 \text{ atm} - 1\text{atm} \quad (\because p_a = 1 \text{ atm})$$

$$p_g = 103 \text{ atm}$$

The pressure outside the submarine while pressure inside, it is p_a.

Given, $A = 400 \text{ cm}^2 = 0.04 \text{ m}^2$.

The net pressure acting on the window is gauge pressure, then $p_g = \dfrac{F}{A}$

$$\Rightarrow \quad F = p_g A$$

or $F = 103 \times 10^5 \times 0.04 = 4.12 \times 10^5$ N

Hydrostatic Paradox

Let us take three vessels A, B and C of different shapes which are open both at the top and the bottom ends. These vessels are connected with a common pipe at the bottom as shown in figure below.

Now, fill the three vessels with the same liquid. The reading, we note that pressure will be same even though the quantity of liquid in different vessels is different.

It means the liquid pressure at a point is independent of the quantity of liquid but depends upon the depth of point below the liquid surface.

This is known as **hydrostatic paradox.**

Three vessels *A*, *B* and *C* containing different amounts of liquids

Hence, the pressure exerted by a liquid depends only on the height of fluid column and is independent of the shape of the containing vessel.

Atmospheric Pressure and Gauge Pressure

The gaseous envelope surrounding the earth is called **earth's atmosphere**. The pressure exerted by the atmosphere is called **atmospheric pressure**. The force exerted by air column of air on a unit area on the earth's surface is equal to the atmospheric pressure, it is denoted by p_a.

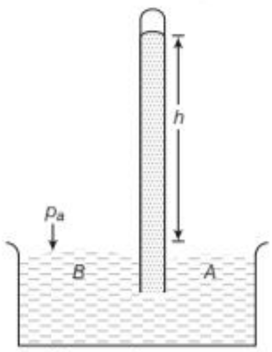
The value of atmospheric pressure on the surface of the earth at sea level called 1 atmosphere (1 atm), it is nearly about $\mathbf{1.013 \times 10^5 N/m^2}$.

The earth's atmosphere exerts a huge pressure, which can be demonstrated by the following methods.

Mercury Barometer

An Italian scientist **Evangelista Torricelli** (1608-1647), first devised an instrument to measure atmospheric pressure. It is known as **barometer**. A long glass tube closed at one end and filled with mercury is inverted into a trough of mercury.

We find that mercury column in the tube has height of about **76 cm** above the mercury level in the trough.



Mercury barometer

If the given tube is inclined or raised up or lowered in mercury trough, the vertical height of mercury level in tube is always found to be constant.

The space above mercury in the tube contains mercury vapour and its pressure can be neglected.

Let us consider points *B* and *A* as shown in the figure. Then, $p_A = p_B$ = atmospheric pressure, p_a.

Let h is the height of mercury column and ρ is the density of mercury,

So, $$p_A = p_B = p_a = h\rho g$$

By putting values atmospheric pressure,

$$h = 0.76\,\text{m}$$
$$\rho = 13.6 \times 10^3\,\text{kg/m}^3$$
$$g = 9.8\,\text{m/s}^2$$
$$p_a = 0.76 \times 13.6 \times 10^3 \times 9.8$$
$$= 1.013 \times 10^5\,\text{N/m}^2$$
$$= 1.013 \times 10^5\,\text{Pa}$$

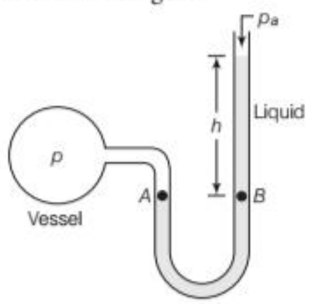
(This is the atmospheric pressure at the sea level).

The pressure can also be stated in terms of cm or mm of mercury (Hg). A pressure equivalent of 1 mm is called a **torr** named after Torricelli.

Open Tube Manometer

Open tube manometer is used to measure pressure difference. It consists of a U-shape tube containing a liquid having low density such as oil to measure the small pressure difference and mercury as the high density liquid to measure the large pressure differences. One end of the tube is open to the atmosphere and the other end is joined with a flask, whose pressure we want to know.

The arrangement is shown in figure.



Open tube manometer

The pressure p at point *A* is equal to pressure at point *B*.

$\therefore$ $$p_A = p_B \text{ (Pascal's law)}$$

As $$p_A = p_a + h\rho g$$

So, $$p_B = p_a + \rho g h$$

Absolute Pressure and Gauge Pressure

The total or actual pressure p at a point is called absolute pressure. Gauge pressure is the difference between the actual pressure (or absolute pressure) at a point and the atmospheric pressure, i.e. $p_g = p - p_a = h\rho g$

The gauge pressure is proportional to h. Many pressure measuring devices directly measure the gauge pressure. These include the tyre pressure gauge and the blood pressure gauge (sphygmomanometer).

EXAMPLE |5| Absolute and Gauge Pressure in a Tank

What is the absolute and gauge pressure of the gas above the liquid surface in the tank shown in figure? Density of oil = 820 kg-m^{-3}, density of mercury $= 13.6 \times 10^3$ kgm^{-3}. Given, 1 atm pressure $= 1.01 \times 10^5$ Pa. **[NCERT]**

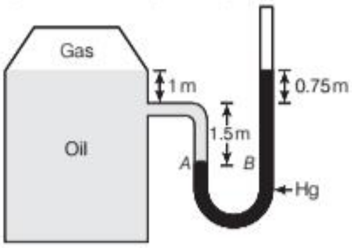
Sol. As the points A and B are at the same level in the mercury column, so $p_A = p_B$

Now, $p_A = p + (1.50 + 1.00) \times 820 \times 9.8$

where, p is the pressure of the gas in the tank.

And $p_B = p' + (1.50 + 0.75) \times 13.6 \times 10^3 \times 9.8$

where, p' is the atmospheric pressure.



As, $p_A = p_B$

$$p - p' = 2.25 \times 13.6 \times 10^3 \times 9.8 - 2.50 \times 820 \times 9.8$$

$$= 3 \times 10^5 - 0.2 \times 10^5 = 2.8 \times 10^5 \text{ Pa}$$

$\therefore$ Gauge pressure

= absolute pressure − atmospheric pressure.

$p_g = p - p' = 2.8 \times 10^5$ Pa $\quad [\because p' = p_a]$

Absolute pressure

= gauge pressure + atmospheric pressure

$p = 2.8 \times 10^5 + 1.01 \times 10^5 = 3.81 \times 10^5$ Pa

Height of Atmosphere

The standard atmospheric pressure is 1.013×10^5 Pa. If the atmosphere of the earth has uniform density $\rho = 1.29$ kg/m^3, then the height h of the air column which exerts the standard atmospheric pressure is given by $h\rho g = 1.013 \times 10^5$

$$h = \frac{1.013 \times 10^5}{\rho g} = \frac{1.013 \times 10^5}{1.29 \times 9.8} \text{ m}$$

$$h = 7.95 \times 10^3 \text{ m} \approx 8 \text{ km}$$

In fact, both the density of air and the value of g decrease with height and the earth's atmosphere extends up to 100 km.

Different Units of Pressure

(i) **Atmosphere** (atm) It is the pressure exerted by 76 cm of Hg column (at 0°C, 95° latitude and mean sea level).

(ii) In meteorology, the atmospheric pressure is measured in bar and millibar. 1 bar

$$= 10^5 \text{ Pa} = 10^6 \text{ dyne/cm}^2$$

$$1 \text{ millibar} = 10^{-3} \text{ bar} = 100 \text{ Pa}$$

(iii) Atmospheric pressure is also measured in torr a unit named after Torricelli.

$$1 \text{ torr} = 1 \text{ mm of Hg} = 133 \text{ Pa}$$

$$1 \text{ atm} = 1.013 \text{ bar} = 760 \text{ torr}$$

Note

- A drop in the atmospheric pressure by 10 mm of Hg or more is a sign of an approaching storm.
- The gauge pressure may be positive or negative depending on $p > p_a$ or $p < p_a$. In inflated tyres or the human circulatory system, the absolute pressure is greater than atmospheric pressure, so gauge pressure is positive, called the **over pressure**.
- However, when we suck a fluid through a straw, the absolute pressure in our lungs is less than atmospheric pressure and so, the gauge pressure is negative.

Pascal's Law for Transmission of Fluid Pressure

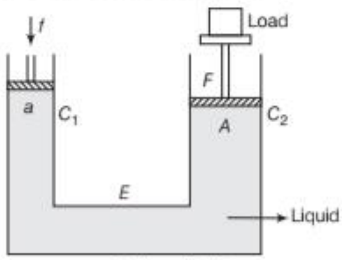
Pascal's law states that, **whenever external pressure is applied on any part of a fluid contained in a vessel at rest, it is transmitted undiminished and equally in all directions.**

This is the Pascal's law for transmission of fluid pressure and has many applications in daily life.

Applications of Pascal's Law

(i) Hydraulic Lift

Hydraulic lift is an application of Pascal's law. It is used to lift heavy loads. It is a force multiplier.



Hydraulic lift

It consists of two cylinders C_1 and C_2 connected to each other by a pipe E. These cylinders are fitted with water-tight frictionless pistons of different cross-sectional areas. These cylinders and the pipe contain a liquid.

Suppose, a downward force f is applied on the smaller piston of cross-sectional area a. Then, pressure exerted on the liquid,

$$p = \frac{f}{a}$$

According to Pascal's law, this pressure p is transmitted to the larger piston of cross-sectional area A, then upward force on larger piston C_2 is

$$F = p \times A = \frac{f}{a} \times A = \frac{A}{a} \times f$$

Upward force on a larger piston, $F = \frac{A}{a} f$

As, $A > a$, therefore, $F > f$.

This shows that the small forces applied on the smaller piston (acting downward) will be appearing as a very large force (acting upward) on the larger piston. As a result of it, a heavy load placed on the larger piston is easily lifted upwards.

EXAMPLE |6| Force Exerted by Larger Piston

Two syringes of different cross-sections (without needles) filled with water are connected with a tightly fitted rubber tube. Diameters of the smaller and larger piston are 1.0 cm and 3.0 cm, respectively.

(i) Find the force exerted on the larger piston when a force of 10 N is applied to the smaller piston.

(ii) If the smaller piston is pushed in through 6.0 cm, how much does the larger piston move out? **[NCERT]**

Sol. (i) Here, $A_1 = \pi \left(\frac{D_1}{2}\right)^2$, $A_2 = \pi \left(\frac{D_2}{2}\right)^2$

$$A_1 = \pi \left(\frac{3}{2} \times 10^{-2}\right)^2 \text{m}^2,$$

$$A_2 = \pi \left(\frac{1}{2} \times 10^{-2}\right)^2 \text{m}^2$$

$\because$ Pressure is transmitted undiminished through water, so,

$$\therefore \quad F_2 = \frac{A_2}{A_1} F_1 = \frac{\pi \left(\frac{3}{2} \times 10^{-2}\right)^2}{\pi \left(\frac{1}{2} \times 10^{-2}\right)^2} \times 10$$

$$\Rightarrow \quad F_2 = 90 \text{ N}$$

(ii) Volume covered by the movement of smaller piston inwards is equal to volume moved outwards due to the larger piston.

$\because$ Water is incompressible

$$\therefore \quad L_1 A_1 = L_2 A_2$$

$$L_2 = \frac{A_1}{A_2} L_1 = \frac{\pi \left(\frac{1}{2} \times 10^{-2}\right)^2}{\pi \left(\frac{3}{2} \times 10^{-2}\right)^2} \times 6 \times 10^{-2}$$

$$L_2 = 0.67 \times 10^{-2} \text{ m} = 0.67 \text{ cm}$$

Atmospheric pressure is common to both pistons and has been ignored.

EXAMPLE |7| Pressure needed to lift a car

In a car lift compressed air exerts a force F_1 on a small piston having a radius of 5.0 cm. This pressure is transmitted to a second piston of radius 15 cm. If the mass of the car to be lifted to 1350 kg, calculate F_1. What is the pressure necessary to accomplish this task? ($g = 9.8 \text{ ms}^{-2}$). **[NCERT]**

Sol. Since, the pressure is transmitted undiminished through the fluid, calculate the force F_2 exerting by the car.

Given, $r_1 = 5$ cm $= 0.05$ m, $r_2 = 15$cm $= 0.15$ m, $m = 1350$ kg

$$F_2 = mg = 1350 \times 9.81 \text{ N}$$

As the pressure through air is transmitted equally in all directions, so that the pressure p_1 and p_2 will be same.

$$p_1 = p_2$$

$$\Rightarrow \quad \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\Rightarrow \quad \frac{F_1}{\pi r_1^2} = \frac{F_2}{\pi r_2^2}$$

$$\therefore \quad F_1 = F_2 \times \left(\frac{r_1}{r_2}\right)^2$$

$$= 1350 \times 9.81 \times \left(\frac{5}{15}\right)^2$$

$$\simeq 1.5 \times 10^3 \text{ N}$$

Now, the required air pressure necessary to lift the car can be calculated.

The air pressure p which will produce the force of 1.5×10^3 N is $p = \frac{F_1}{A_1} = \frac{1.5 \times 10^3}{\pi (0.05)^2} \Rightarrow p = 1.9 \times 10^5$ Pa

EXAMPLE |8| Work Done by Hydraulic Lift

Consider the hydraulic lift having small cylindrical piston and larger piston with radii 4.0 cm and 16 cm, respectively. If a truck of mass 3000 kg is placed on the larger piston, then find out the force must be applied to the small piston to lift the truck. Also find how far must the small piston move down to lift the truck through a height of 0.20 m. Assume initially these two pistons are at the same height. (Take $g = 10 \text{ m/s}^2$)

A downward force F_1 is applied to a piston with a small area or smaller piston and a downward force with magnitude F_2 equal to the weight of the truck is applied on the larger piston.

Sol. Given, $r_1 = 4.0 \text{ cm} = 0.04 \text{ m}$,

$r_2 = 16.0 \text{ cm} = 0.16 \text{ m}$,

$m = 3000 \text{ kg}, d_2 = 0.20 \text{ m}$

Force F_2 = weight of the truck $= mg = 3000 \times 10$

$F_2 = 30000 \text{N}$

From Pascal's law, the pressure at given level is the same throughout the hydraulic fluid. Since, the piston are at the same level. Therefore, the downward force on the smaller piston can be determined as

$$F_1 = \frac{A_1}{A_2} \times F_2$$

where, $A_1 = \pi r_1^2 = \pi(0.04)^2 = 0.005024$

$$A_2 = \pi r_2^2 = \pi(0.16)^2 = 0.080384 \text{ m}^2$$

$$F_1 = \frac{0.005024}{0.080384} \times 30000 = 1875 \text{ N}$$

Hence, a force of 1875 N on small piston can lift a truck weighing 30000 N.

Now, apply the conservation of energy as if we neglect friction, then the amount of work done by the applied force must equal to the work done to lift the truck. Thus,

$$W_1 = W_2$$

$$\Rightarrow \quad F_1 d_1 = F_2 d_2 \quad [\because \text{ given, } d_2 = 0.20 \text{ m}]$$

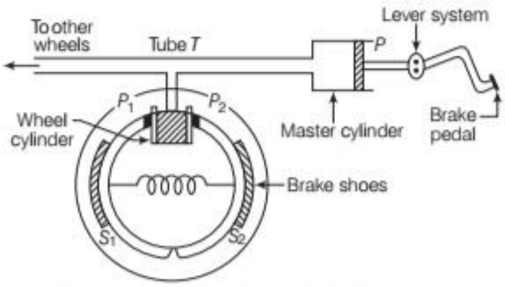
$$1875 \times d_1 = 30000 \times 0.20$$

$$d_1 = \frac{30000 \times 0.20}{1875} = 3.2 \text{m}$$

(ii) Hydraulic Brakes

The working of the hydraulic brakes is also based on the Pascal's law.

It consists of a tube T containing brake oil. One end of the tube is connected to the wheel cylinder having two pistons P_1 and P_2. The pistons P_1 and P_2 are connected to the brake shoes S_1 and S_2, respectively and the other end of tube is connected to master cylinder fitted with the piston P.



Constructional details of hydraulic brakes

The piston P is connected to the brake pedal through the lever system. The area of cross-section of the wheel cylinder is greater than that of master cylinder. The arrangement is shown in above figure.

Working of Hydraulic Brakes

(i) When the brake pedal is pressed, the lever system operates. The piston P is pushed.

The pressure is transmitted to pistons P_1 and P_2 in accordance with Pascal's law.

(ii) The pistons (P_1 and P_2) push the brake shoes away from each other. The brake shoes press against the inner rim of the wheel and retard the motion of the wheel.

(iii) Since, the area of the pistons of the wheel cylinder is greater than the area of piston P, therefore, a small force applied to the brake pedal produces a large thrust on the wheels and hence, the brake becomes operative.

(iv) When the pressure on the brake pedal is released, the brake shoes return to their normal pistons by the action of spring (i.e., it pulls the brake shoes away from the rim) which in turn force back the oil from wheel cylinder into the master cylinder.

(v) In order to apply equal pressure to all the wheels, the master cylinder is connected to all the wheels of the vehicle through tubes.

(vi) Hence, small force applied to the pedal exerts a much larger force on the wheel drums, which enables the driver to keep the vehicle under control.

BUOYANCY

When a body is partially or wholly immersed in a fluid at rest, it displaces the fluid. The displaced fluid exerts a thrust or an upward force on the body.

The upward force acting on the body immersed in a fluid is called **upward thrust** or **buoyant force** and the phenomenon is called **buoyancy**.

We know the weight of the body acts at its centre of gravity. But the buoyant force acts at the **centre of buoyancy** which is the centre of gravity of the liquid displaced by the body, when immersed in the liquid.

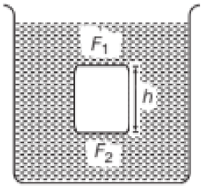
ARCHIMEDES' PRINCIPLE

The Archimedes' principle gives the magnitude of buoyant force on a body. It states that

when a body is immersed wholly or partially in a liquid at rest, it experiences an upthrust. The upthrust is equal to the weight of the liquid displaced by the immersed part of the body and its upthrust acts through the centre of gravity of the displaced fluid.

Explanation of Archimedes ' Principle

Consider a cylindrical body of height h immersed in a fluid. The upward force (F_2) on the bottom of the body is more than the downward force (F_1) on its top. If p_1 and p_2 are the pressures at upper face and lower face of the body, respectively.



The resultant upward force on the body exerted by the fluid is equal to $(p_2 - p_1)A$.

As we know, $p_2 - p_1 = \rho g h$

So, $(p_2 - p_1)\, A = h\rho g A = mg$ $\quad [\because m = Ah\rho]$

where, ρ = density of fluid and

A = cross-sectional area of the body

Thus, the upward force exerted by the fluid on the body is equal to the weight of the displaced fluid.

The following three cases are possible when the body is immersed in the fluid.

(i) If the weight of the body is greater than the upward force or upthrust by the fluid acting upwards, then the body sinks.

(ii) If the weight of the body is equal to the upthrust or the weight of the body is just balanced by upthrust, then the body floats fully immersed.

(iii) If the weight of the body is less than the upward force, then the body floats partly immersed.

If the total volume of the body is V_S and a part V_P of it is submerged in the fluid, then at equilibrium,

Weight of the body = Weight of fluid displaced

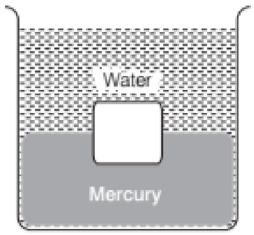
$$V_S \rho_S g = V_P \rho_l g$$

where, ρ_S and ρ_l are the densities of the body and fluid, respectively.

$$\boxed{\frac{\rho_S}{\rho_l} = \frac{V_P}{V_S}}$$

EXAMPLE |9| A Cube Placed in Water and Mercury

A tank contains water and mercury as shown in figure. An iron cube of edge 6 cm is in equilibrium as shown in figure. What is the fraction of cube inside the mercury? Given, density of iron = 7.7×10^3 kgm^{-3} and density of mercury = 13.6×10^3 kgm^{-3}.



Sol. Let y be the depth of cube in mercury. The depth of cube in water will be $(0.06 - y)$ m. The buoyant force on cube due to mercury can be find out by using Archimedes' principle.

$$B_1 = (0.06)^2 \times y \times (13.6 \times 10^3) \times 9.8 \text{ N}$$

Similarly, due to water the buoyant force on cube is

$$B_2 = (0.06)^2 \times (0.06 - y) \times 10^3 \times 9.8 \text{ N}$$

When cube is in equilibrium, the sum of both the buoyant forces will be equal to the total weight of iron cube. $B_1 + B_2$ = weight of iron cube

or $\quad (0.06)^2 \times 10^3 \times 9.8\, [13.6\, y + (0.06 - y)]$

$$= (0.06)^3 \times 7.7 \times 10^3 \times 9.8$$

$$\Rightarrow \quad y = \frac{0.396}{12.6} = 0.032 \text{ m}$$

Fraction of cube inside mercury $= \dfrac{0.032}{0.06}$

$$= 0.533 = 53\%$$

EXAMPLE |10| Up and Down Motion of a Fish

What is the principle behind the up and down motion of a fish in water?

Sol. An object can float on water if its density is less than water. A fish can adjust its density by expanding or contracting itself. The fish can move upward by decreasing its density and downward by increasing its density.

Note

When ice floats in a liquid and completely melts, then there could be following cases

(i) If $\rho_l > \rho_w$, the level of liquid will rise.

(ii) If $\rho_l < \rho_w$, the level of liquid decreases.

(iii) If $\rho_l = \rho_w$, the level of liquid will remain the same.

TOPIC PRACTICE 1

OBJECTIVE Type Questions

1. Average pressure p_{av} is defined as

(a) $p_{av} = \frac{F}{A}$ (b) $p_{av} = \frac{V}{F}$

(c) $p_{av} = \frac{A}{F}$ (d) $p_{av} = \frac{F}{V}$

Sol. (a) If F is the magnitude of the normal force acting over an area A, then the average pressure p is defined as the normal force acting per unit area.

$$p_{av} = \frac{F}{A}$$

2. The density of water at 4°C is

(a) 1.0×10^3 kgm^{-3} (b) 4×10^2 kgm^{-3}

(c) 6×10^3 kgm^{-3} (d) 3.2×10^3 kgm^{-3}

Sol. (a) The density of water at 4°C (277 K) is 1.0×10^3 kgm^{-3}.

3. Pressure at a point inside a liquid does not depend on

(a) the depth of the point below the surface of the liquid.

(b) the nature of the liquid.

(c) the acceleration due to gravity at that point.

(d) total weight of fluid in the beaker.

Sol. (d) $p = \frac{F}{A} = \rho g h$

$\therefore$ It does not depend on the weight of fluid.

4. Three liquids of densities d, $2d$ and $3d$ are mixed in equal proportion of weights. If density of water is d, then the specific gravity of the mixture is

(a) $\frac{11}{7}$ (b) $\frac{18}{11}$ (c) $\frac{13}{9}$ (d) $\frac{23}{18}$

Sol. (b) $w_1 = w_2 = w_3$

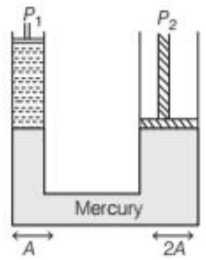
$\Rightarrow$ $m_1 = m_2 = m_3 = m$ (say)

Then, $V_1 = \frac{m}{d}, V_2 = \frac{m}{2d}, V_3 = \frac{m}{3d}$

$\therefore$ $d_{mix} = \frac{\text{Mass}}{\text{Volume}} = \frac{3m}{V_1 + V_2 + V_3} = \frac{18}{11}d$

So, specific gravity of mixture $= \frac{d_{mix}}{d_{water}} = \frac{18}{11}$

5. A hydraulic lift has 2 limbs of areas A and $2A$. Force F is applied over limb of area A to lift a heavy car. If distance moved by piston P_1 is x, then distance moved by piston P_2 is



(a) x (b) $2x$ (c) $\frac{x}{2}$ (d) $4x$

Sol. (c) As, $V_1 = V_2 \Rightarrow A_1x_1 = A_2x_2$

$\Rightarrow$ $x_2 = \frac{A_1}{A_2}x_1 = \frac{Ax}{2A} = \frac{x}{2}$

VERY SHORT ANSWER Type Questions

6. Three vessels have same base area and different neck area. Equal volume of liquid is poured into them, which will possess more pressure at the base?

Sol. If the volumes are same, then height of the liquid will be highest in which the cross-sectional area is least at the top. So, the vessel having least cross-sectional area at the top possess more pressure at the base ($\because p = \rho g h$).

7. What is the use of open tube manometer?

Sol. Open tube manometer is used for measuring pressure difference.

8. A mercury barometer is placed in the mercury trough in a way that the angle made with the vertical is 60°. Find the height of mercury column.

Sol. The height of mercury in the inclined case will be 76 cm.

9. What is gauge pressure?

Sol. The difference between absolute pressure and atmospheric pressure is known as gauge pressure.

As, $p_{absolute} = p_a + \rho g h$

So, $p_{absolute} - p_a = \rho g h$ i.e. $p_{gauge} = \rho g h$

Here, ρ is the density of a fluid of depth h.

10. A vessel filled with water is kept on a weighing pan and the scale adjusted to zero. A block of mass M and density ρ is suspended by a massless spring of spring constant k.

This block is submerged inside into the water in the vessel. What is the reading of the scale? [NCERT]

Sol. The scale is adjusted to zero, therefore, when block suspended to a spring is immersed in water, then the reading of the scale will be equal to the thrust on the block due to water.

$$\text{Thrust} = V\rho_w g = \frac{m}{\rho}\rho_w g \qquad \left[\because V = \frac{m}{\rho}\right]$$

11. How will a mercury barometer's height change, if we introduce some water vapour into it.

Sol. Because water vapour exerts pressure. Thus, the barometric height decreases.

SHORT ANSWER Type Questions

12. A container of area $0.02\ m^2$ is filled with water. Find the pressure at the bottom of the container if a weight is placed on the piston. (as shown in the figure).

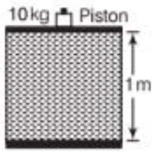
Sol. The net force acting on the base

$$= (\rho g h)A + mg$$

$$F = 1000 \times 9.8 \times 1 \times 0.02 + 10 \times 9.8$$

$$= 294\ N$$

$$\therefore \text{ Pressure} = \frac{\text{Thrust}}{\text{Area}} = \frac{294\ N}{0.02} = 14700 N$$



13. A 50 kg girl wearing high heels shoes balances on a single heel. The heel is circular with a diameter 1.0 cm. What is the pressure exerted on the horizontal floor? **[NCERT]**

Sol. Given, mass of girl $(m) = 50$ kg

Diameter of circular heel $(2r) = 1.0$ cm

$\therefore$ Radius $(r) = 0.5$ cm $= 5 \times 10^{-3}$ m

Area of circular heel $(A) = \pi r^2 = 3.14 \times (5 \times 10^{-3})^2 m^2$

$$= 78.50 \times 10^{-6} m^2$$

$\therefore$ Pressure exerted on the horizontal floor

$$p = \frac{F}{A} = \frac{mg}{A} = \frac{50 \times 9.8}{78.50 \times 10^{-6}} = 6.24 \times 10^6 \text{ Pa}$$

14. If the required pressure in the tyre of a car is 199 kPa, then

(i) what is the gauge pressure?

(ii) what is the absolute pressure?

Sol. (i) Gauge pressure

$\therefore$ $p_g = 199$ kPa

(ii) Absolute pressure

$$p = p_a + p_g \quad [\because \text{from Eq. (i)}]$$

$$= 101 \text{ kPa} + 199 \text{ kPa} = 300 \text{ kPa}$$

15. A hydraulic automobile lift is designed to lift cars with a maximum mass of 3000 kg. The area of cross-section of the piston carrying the load is $425\ cm^2$. What maximum pressure would the smaller piston have to bear? **[NCERT]**

Sol. Given, maximum mass that can be lifted $(m) = 3000$ kg

Area of cross-section $(A) = 425\ cm^2 = 4.25 \times 10^{-2} m^2$

$\therefore$ Maximum pressure on the bigger piston

$$p = \frac{F}{A} = \frac{mg}{A} = \frac{3000 \times 9.8}{4.25 \times 10^{-2}} = 6.92 \times 10^5 \text{ Pa}$$

According to Pascal's law, the pressure applied on an enclosed liquid is transmitted equally in all directions.

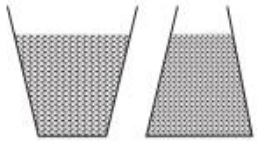
$\therefore$ Maximum pressure on smaller piston

= maximum pressure on bigger piston

$$p' = p = 6.92 \times 10^5 \text{ Pa}$$

16. Two vessels have the same base area but different shapes. The first vessel takes twice the volume of water than the second vessel required to fill upto a particular common height. Is the force exerted by the water on the base of the vessel the same in the two cases? If so, why do the vessels filled with water to that same height give different readings on a weighing scale? **[NCERT]**

Sol. The pressure exerted by a liquid column depends upon its height. The height of water in both vessels are same, therefore, the pressure on the base of each vessel will be same.



The area of the base of each vessel is also same and hence, force exerted by the water on the base of the vessels is same. Liquid also applies force on the walls of the vessel. As walls of the vessels are not perpendicular to the base, therefore, force exerted on the walls by the liquid has a non-zero vertical component, which is more in first vessel. Therefore, the two vessels filled with water to the same height give different readings on a weighing scale.

17. Torricelli's barometer used mercury. Pascal duplicated it using French wine of density $948\ kg/m^3$. Determine the height of the wine column for normal atmospheric pressure. **[NCERT]**

The pressure exerted by a liquid column

where, $p = h\rho g$, h = height of liquid column

ρ = density of liquid

g = acceleration due to gravity.

Sol. Atmospheric pressure $(p) = 1.013 \times 10^5$ Pa

Density of French wine $(\rho) = 984\ kg/m^3$

Let h be the height of wine column,

$$p = h\rho g, \quad h = \frac{p}{\rho g} = \frac{1.013 \times 10^5}{984 \times 9.8} = 10.5 \text{ m}$$

18. A vertical off-shore structure is built to withstand a maximum stress of 10^9 Pa. Is the structure suitable for putting up on top of an oil well in the ocean? Take the depth of the ocean to be roughly 3 km and ignore ocean currents. **[NCERT]**

Sol. Given, depth of ocean $(h) = 3\text{ km} = 3000\text{ m}$

Density of water $(\rho) = 10^3\text{ kg/m}^3$

Pressure exerted by water column

$$p = h\rho g = 3000 \times 10^3 \times 9.8$$
$$= 29.4 \times 10^6 = 294 \times 10^7\text{ Pa}$$

Maximum stress which can be withstand by the vertical off-shore structure $= 10^9$ Pa

As, $\quad 10^9\text{ Pa} > 2.9 \times 10^7\text{ Pa}$

Therefore, the vertical structure is suitable for putting up on top of an oil well in the ocean.

19. A balloon with hydrogen in it rises up but a balloon with air comes down, Why?

Sol. The density of hydrogen is less than air. So, the buoyant force on the balloon will be more than its weight in case of the hydrogen. So, in this case the balloon rises up. In case of air, the weight of balloon is more than the buoyant force acting on it, so balloon will come down. **[2]**

20. A body of mass 6 kg is floating in a liquid with 2/3 of its volume inside the liquid. Find ratio between the density of the body and density of liquid. Take $g = 10\text{m/s}^2$.

Sol. As we know that, for a floating body

Buoyant force = Weight of liquid displaced

Let V be the volume of the body $\frac{2}{3}V\rho_l g = V\rho_l g$

where, ρ_b = density of floating body

and $\quad \rho_l$ = density of liquid

$$\therefore \quad \frac{\rho_b}{\rho_l} = \frac{2}{3}$$

LONG ANSWER Type I Questions

21. Explain why?

(i) The blood pressure in humans is greater at the feet than the brain.

(ii) Atmospheric pressure at a height of about 6 km decreases to nearly half its value at the sea level though the height of the atmosphere is more than 100 km.

(iii) Hydrostatic pressure is a scalar quantity even though pressure is force divided by area. **[NCERT]**

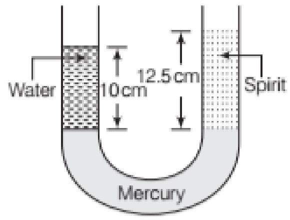
Sol. (i) The pressure of liquid column is given by $p = h\rho g$, where h is depth, ρ is density and g is acceleration due to gravity.

Therefore, pressure of liquid column increases with depth. The height of blood column in human body is more at feet than at the brain. Therefore, the blood pressure in humans is greater at the feet than the brain.

(ii) The density of air is maximum near the surface of the earth and decreases rapidly with height. At a height of 6 km, the density of air decreases to nearly half of its value at the sea level. Beyond 6 km height, the density of air decreases very slowly with height. Hence, the atmospheric pressure at a height of about 6 km decreases to nearly half of its value at the sea level.

(iii) When force is applied on a liquid, the pressure is transmitted equally in all directions inside the liquid. Therefore, hydrostatic pressure has no fixed direction and hence, it is a scalar quantity.

22. A U-tube contains water and methylated spirit separated by mercury. The mercury columns in the two arms are in level with 10.0 cm of water in one arm and 12.5 cm of spirit in the other. What is the specific gravity of spirit? **[NCERT]**



Sol. Height of water column, $h_1 = 10.0$ cm

Density of water, $\rho_1 = 1\text{ g/cm}^3$

Height of spirit column, $h_2 = 12.5$ cm

Density of spirit, $\rho_2 = ?$

The mercury column in both arms of the U-tube are at same level, therefore, pressure in both arms will be same.

$\therefore$ Pressure exerted by water column = pressure exerted by spirit column

$$\therefore \quad p_1 = p_2$$
$$h_1\rho_1 g = h_2\rho_2 g$$

or $\quad \rho_2 = \frac{h_1\rho_1}{h_2} = \frac{10 \times 1}{12.5} = 0.80\text{ g/cm}^3$

Specific gravity of spirit $= \frac{\text{Density of spirit}}{\text{Density of water}}$

$$= \frac{0.80}{1} = 0.80$$

23. In the previous question, if 15.0 cm of water and spirit each are further poured into the respective arms of the tube, what is the difference in the levels of mercury in the two arms? (specific gravity of mercury = 13.6) **[NCERT]**

Sol. When 15.0 cm of water is poured in each arm, then

Height of water column $(h_1) = 10 + 15 = 25$ cm

Height of spirit column $(h_2) = 12.5 + 15 = 27.5$ cm
Density of water $(\rho_w) = 1\ \text{g/cm}^3$
Density of spirit $(\rho_s) = 0.80\ \text{g/cm}^3$
Density of mercury $(\rho_m) = 13.6\ \text{g/cm}^3$
Let in equilibrium, the difference in the level of mercury in both arms be h cm.

$\therefore\quad h\rho_m g = h_1\rho_w g - h_2\rho_s g$

or $\quad h = \dfrac{h_1\rho_w - h_2\rho_s}{\rho_m} = \dfrac{25 \times 1 - 27.5 \times 0.80}{13.6}$

$= 0.221$ cm

Therefore, mercury will rise in the arm containing spirit by 0.221 cm.

24. During blood transfusion, the needle is inserted in a vein where the gauge pressure is 2000 Pa. At what height must the blood container be placed so that blood may just enter the vein? (Use the density of whole blood from table). **[NCERT]**

Densities of Some Common Fluids at STP

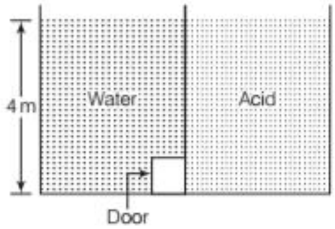
Fluid	Density ρ(gm^{-3})
Water	1.00×10^3
Sea water	1.03×10^3
Mercury	13.6×10^3
Ethyl alcohol	0.806×10^3
Whole blood	1.06×10^3
Air	1.29
Oxygen	1.43
Hydrogen	9.0×10^{-2}
Interstellar space	$\approx 10^{-22}$

Sol. Given, gauge pressure $(p) = 2000$ Pa
Density of whole blood $(\rho) = 1.06 \times 10^3\ \text{kg/m}^3$
$g = 9.8\ \text{m/s}^2$
Let the container be placed at height h metre.
Pressure exerted by the blood $(p) = h\rho g$

or $\quad h = \dfrac{p}{\rho g} = \dfrac{2000}{1.06 \times 10^3 \times 9.8}$

$= 0.193\ \text{m} \simeq 0.20$ m

25. A tank with a square base of area 1.0 m^2 is divided by a vertical partition in the middle. The bottom of the partition has a small hinged door of area 20 cm^2. The tank is filled with water in one compartment and an acid (of relative density 1.7) in the other, both to a height of 4.0. Compute the force necessary to keep the door close. **[NCERT]**



Sol. Given, height of water and acid $(h) = 4.0$ m
Density of water $(\rho_w) = 1 \times 10^3\ \text{kg/m}^3$
Density of acid $(\rho_a) = 1.7 \times 10^3\ \text{kg/m}^3$
Pressure exerted by water at the door

$p_1 = h\rho_w g = 4 \times 1 \times 10^3 \times 9.8$
$= 39.2 \times 10^3$ Pa

Pressure exerted by acid at the door

$p_2 = h\rho_a g = 4 \times 1.7 \times 10^3 \times 9.8$
$= 66.64 \times 10^3$ Pa

$\therefore$ Difference in pressure $(\Delta p) = p_2 - p_1$
$= 66.64 \times 10^3 - 39.2 \times 10^3$
$= 27.44 \times 10^3$ Pa

Area of the door $(A) = 20\ \text{cm}^2$
$= 20 \times 10^{-4}\ \text{m}^2$

Force acting on the door $= \Delta p \times A$ $\quad \left[\because \text{Pressure} = \dfrac{\text{force}}{\text{area}}\right]$

$= 27.44 \times 10^3 \times 20 \times 10^{-4}$
$= 54.88\ \text{N} = 55$ N

Therefore, 55 N force is necessary to keep the door close.

ASSESS YOUR TOPICAL UNDERSTANDING

OBJECTIVE Type Questions

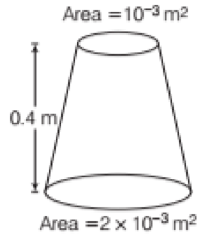
1. The key property of fluids is that
   (a) they offer very little resistance to shear stress
   (b) their shape changes
   (c) they offer very large resistance to shear stress
   (d) Both (a) and (b)

2. The two thin bones (femurs), each of cross-sectional area 10 cm^2 support the upper part of human body of mass 40 kg. Estimate the average pressure sustained by the femurs.
   (a) 2×10^5 Nm^{-2} (b) 3×10^4 Nm^{-2}
   (c) 2.5×10^3 Nm^{-2} (d) 6×10^4 Nm^{-2}

3. If two liquids of same volume but different densities ρ_1 and ρ_2 are mixed, then density of mixture is given by
   (a) $\rho=\frac{\rho_1+\rho_2}{2}$ (b) $\rho=\frac{\rho_1+\rho_2}{2\rho_1\rho_2}$
   (c) $\rho=\frac{2\rho_1\rho_2}{\rho_1+\rho_2}$ (d) $\rho=\frac{\rho_1\rho_2}{\rho_1+\rho_2}$

4. Pressure is applied to an enclosed fluid. It is
   (a) increased and applied to every part of the fluid
   (b) diminished and transmitted to the walls of the container
   (c) increased in proportion to the mass of the fluid and then transmitted
   (d) transmitted unchanged to every portion of the fluid and the walls of container

5. A uniformly tapering vessel is filled with a liquid of density 900 kgm^{-3}. The force that acts on the base of the vessel due to the liquid is (Take, $g = 10\, ms^{-2}$)



   (a) 3.6 N (b) 7.2 N (c) 9.0 N (d) 14.4 N

Answers

1. (d)	2. (a)	3. (a)	4. (d)	5. (b)

VERY SHORT ANSWER Type Questions

6. The blood pressure in humans is greater at the feet than at the brain, why?

7. What will be the following changes in the height of mercury in a barometer indicates?
   (i) Increase (ii) Decrease

8. The thrust due to atmospheric pressure on us is around 15 tonne. How we withstand such a thrust?

SHORT ANSWER Type Questions

9. An ice cube floats in water. What will happen to the water level if the complete ice melts?

10. A block floats in water and this system is placed in an elevator. If the elevator starts accelerating downwards, will the block float with higher or lower immersed part in the water?

11. A dimension of base of vessel is 20 cm $\times$ 10 cm. Water is poured upto a height 5 cm. What is the pressure at the bottom of the vessel? Take $g = 10\ ms^{-2}$.

12. Explain, why atmospheric pressure at a height about 6 km decreases to nearly half of its value at the sea level, though the height of the atmosphere is more than 100 km.

LONG ANSWER Type I Questions

13. A vertical off-shore structure is built to withstand a maximum stress of 10^{10} Pa. Is the structure suitable for putting up on top of an oil well in the ocean? Take the depth of the ocean to be roughly 31 km and ignore ocean currents.

14. A body floats in water with one-third of its volume outside water. While floating in another liquid its $\frac{3}{4}$th volume outside. Find out the density of other liquid. (given density of water = 1 g/cc)

15. If a column of 60 cm height of water supports a 45 cm column of an unknown liquid. Calculate the density of the liquid.
(given density of water = $10^3\, kg/m^3$)

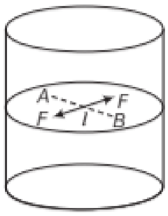
|TOPIC 2|
Surface Tension and Surface Energy

SURFACE TENSION

Surface tension arises due to the fact that the free surface of a liquid at rest has some additional potential energy. Due to it, a liquid surface tends to occupy a minimum surface area and behaves like stretched membrane.

e.g. A steel needle may be made to float on water, though the steel is more dense than water. This is because the water surface acts as a stretched elastic membrane and supports the needle. This property of a liquid is called **surface tension.**

Consider a line *AB* on the free surface of a liquid. The small elements of the surface on this line are in equilibrium because they are acted upon by equal and opposite forces, acting perpendicular to the line from either side as shown in figure.



Definition of surface tension

The force acting on this line is proportional to the length of this line. If l is the length of imaginary line and F the total force on either side of the line, then

$$F \propto l \quad \Rightarrow \quad F = Sl$$

$$S = \frac{F}{l}$$

or $$\boxed{\text{Surface tension, } S = \frac{\text{Force}}{\text{Length}}}$$

From this expression, **surface tension can be defined as the force acting per unit length of an imaginary line drawn on the liquid surface, the direction of force being perpendicular to this line and tangential to the liquid surface.** It is denoted by S and it is a **scalar quantity.**

Units and Dimension of Surface Tension

SI units of surface tension = N/m

CGS unit of surface tension = dyne/cm

$$\text{Dimension of surface tension} = \frac{\text{Force}}{\text{Length}} = \frac{[\text{MLT}^{-2}]}{[\text{L}]}$$
$$= [\text{ML}^0\text{T}^{-2}]$$

Factors Affecting Surface Tension

1. **Temperature** The surface tension of liquid decreases with rise in temperature and *vice-versa.*

   The surface tension of a liquid becomes zero at a particular temperature, called 'Critical temperature' of that liquid. For small temperature differences, surface tension decreases almost linearly as

   $$S_t = S_0(1 - \alpha t)$$

   where, S_t : Surface tension at t°C,
   S_0 : Surface tension at 0°C and
   (α : the temperature sufficient of surface tension)

   Due to this,
   (i) Hot soup taste better than cold soup as hot soup spread over a large area of tongue.
   (ii) Machinery parts get jammed in winter as surface tension of lubricating oil increases with decrease in temperature.

2. **Addition of Impurities** The surface tension of liquids changes appreciably with addition of impurities.(a) A highly soluble substance like sodium chloride increases the surface tension of water. (b)A sparingly soluble substance like phenol or soap reduces the surface tension of water.

Detergent and Surface Tension

Surface tension has a wide use in daily life. The detergents, used for cleaning the dirty clothes in our home is a very good example of surface tension.

Actually, water cannot wet oil stain on your clothes, that is why water alone cannot remove dirt from your clothes.

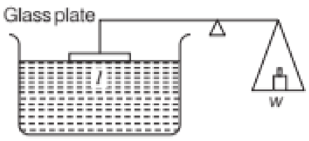
The molecule of detergent can get attached with water and dirt molecules and they take away the dirt with them when we wash the clothes with detergent.

Applications of Surface Tension

(i) Rain drops and drops of mercury placed on glass plate are spherical.
(ii) Hair of shaving brush/painting brush, when dipped in water spread out, but as soon as, it is taken out, its hair stick together.
(iii) A greased needle placed gently on the free surface of water in a beaker does not sink.
(iv) Oil drop spreads on cold water but does not change shape on hot water.

Measuring the Surface Tension of a Liquid

The surface tension of liquid can be measured experimentally as shown in figure. A flat vertical glass plate, below which a vessel filled with some liquid is kept. The plate is balanced by weights on the other side. The vessel is raised slightly until the liquid touches the glass plate and pulls it down because of the force of surface tension. Weights are added till the plate just detaches from water.



Measuring the surface tension

Suppose the additional weight required is mg.

$$S = (mg/2l)$$

where, m is the extra mass and l is the length of the plate edge.

SURFACE ENERGY

The free surface of a liquid always has a tendency to contract and possess minimum surface area. If it is required to increase the surface area of the liquid at a constant temperature, work has to be done. This **work done** is stored in the surface film of the liquid as its **potential energy**.

This potential energy per unit area of the surface film is called the **surface energy**.

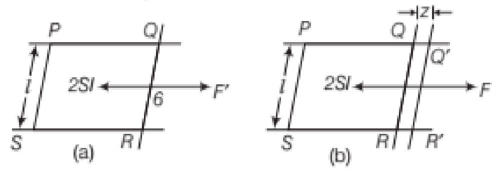
Hence, the surface energy may be defined as the amount of work done in increasing the area of the liquid surface by unity. Thus,

$$\text{Surface energy} = \frac{\text{Work done in increasing the surface area}}{\text{Increase in surface area}}$$

The SI unit of surface energy is J/m^2.

Relation between Surface Energy and Surface Tension

Consider a rectangular frame $PQRS$. Here, wire QR is movable. A soap film is formed on the frame. The film pulls the movable wire QR inward due to surface tension.



Equilibrium position of the film

Stretched position of the film

As, $\text{Surface tension} = \dfrac{\text{force}}{\text{length}} = \dfrac{F'}{2l}$

$$F' = S \times 2l$$

If QR is moved through a distance z by an external force F very slowly, then some work has to be done against this force.

$\therefore$ External work done = Force $\times$ distance

$$= S \times 2l \times z \quad [\because F' = F]$$

Increase in surface area of film $= 2l \times z$

[As soap film has two sides]

$$\text{Surface energy} = \frac{\text{Work done}}{\text{Surface area}} = \frac{S2lz}{2lz} = S$$

So, **value of surface energy of liquid is numerically equal to the value of surface tension.**

ANGLE OF CONTACT

The surface of liquid near the plane of contact with another medium is in curved shape.

The angle between tangent to the liquid surface at the point of contact and the solid surface inside the liquid is called as angle of contact. It is denoted by θ.

The value of angle of contact depends on the following factors

(i) Nature of the solid and liquid in contact.

(ii) Cleanliness of the surface in contact.

(iii) Medium above the free surface of the liquid.

(iv) Temperature of the liquid

e.g. Water forms droplets on lotus leaf shown in Fig. (a) while spreads over a clean plastic plate in Fig. (b).

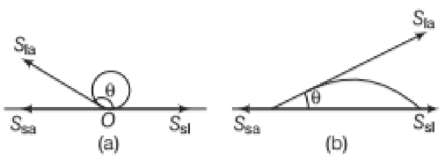
Consider the three interfacial tensions at all the three interfaces such as

S_{sa} = surface tension between solid and air

S_{la} = surface tension between liquid and air

S_{sl} = surface tension between solid and liquid

At the line of contact, the surface forces between the three media must be in equilibrium. Resolving S_{la} into two rectangular components, we have $S_{la}\cos\theta$ acts along the solid surface and $S_{la}\sin\theta$ acts along the perpendicular to the solid surface.



Different shapes of water drops with interfacial tensions

As the liquid on the surface of soild is at rest, so the molecules of these interfaces are in equilibrium. Thus, the net force on them is zero.

For the molecule O to be in equilibrium.

$$S_{sl} + S_{la} \cos\theta = S_{sa}$$

$$\boxed{\cos\theta = \frac{S_{sa} - S_{sl}}{S_{la}}}$$

The following cases arise

(i) If the surface tension at the solid-liquid S_{sl}, interface is greater than the surface tension at the liquid-air S_{la} interface, i.e. $S_{sl} > S_{la}$, then $\cos\theta$ is negative and $\theta > 90°$ (the angle of contact is **obtuse angle**). The molecules of a liquid are attracted strongly to themselves and weakly to those of solid. It costs a lot of energy to create a liquid-solid surface. The liquid then does not wet the solid.

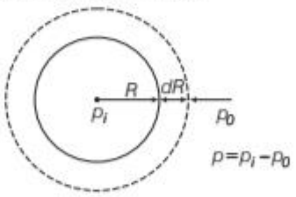
e.g. Water-leaf or glass-mercury interface.

(ii) If the surface tension at the solid-liquid S_{sl} interface is less than the surface tension at the liquid-air S_{la} interface, i.e. $S_{sl} < S_{la}$, then $\cos\theta$ is positive and $\theta < 90°$ (the angle of contact is **acute angle**). The molecules of the liquid are strongly attracted to those of solid and weakly attracted to themselves. It costless energy to create a liquid-solid surface and liquid wets the solid.

e.g. When soap or detergent is added to water, the angle of contact becomes small.

Excess Pressure Inside a Liquid Drop

Suppose a spherical liquid drop of radius R and S be the surface tension of liquid. Due to its spherical shape, there is an excess pressure p inside the drop over that on outside. This excess pressure acts normally outwards. Due to this pressure, radius increases from R to $R + dR$, then extra surface energy can be determined.



Excess pressure inside a liquid drop

Excess pressure inside the drop, $p = p_i - p_0$

where, p_i = total pressure inside the liquid drop

p_0 = atmospheric pressure

Initial surface area of the liquid $= 4\pi R^2$

Final surface area of the liquid drop

$$= 4\pi (R + dR)^2$$
$$= 4\pi (R^2 + 2R\,dR + dR^2)$$
$$= 4\pi R^2 + 8\pi R dR$$

[dR^2 is very small and hence neglected]

Increase in the surface area of liquid drop

$$= 4\pi R^2 + 8\pi R\,dR - 4\pi R^2$$
$$= 8\pi R dR$$

External work done in increasing the surface area of the drop

= Increase in surface energy

= Increase in surface area × Surface tension

$$= (8\pi R dR) \times S \quad \text{...(i)}$$

But work done

= Excess pressure × Area × Change in radius

$$= p \times 4\pi R^2 \times dR \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

So, $$p \times 4\pi R^2 \times dR = 8\pi R dRS$$

$$\boxed{\text{Excess pressure, } p = \frac{2S}{R}}$$

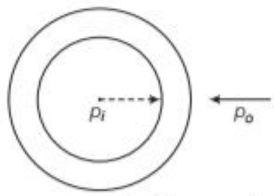
So, **Pressure difference in a drop bubble,**

$$p_i - p_0 = \frac{2S}{R}$$

Excess Pressure Inside a Soap Bubble

From the above case, Increase in surface area $= 8\pi R dR$

But a soap bubble has two free surfaces.



Excess pressure inside a soap bubble

So, effective increase in surface area of the soap bubble

$$= 2 \times 8\pi R\,dR = 16\pi R\,dR \quad \text{...(i)}$$

External work done in increasing the surface area of the soap bubble = Increase in surface energy

= Increase in surface area × surface tension

$$= 16\pi R\,dRS$$

But, work done = Force × change in radius

where, Force = Excess pressure × area $= (p \times 4\pi r^2)$

So, work done $= p \times 4\pi R^2 \times dR$ ...(ii)

From Eqs. (i) and (ii), we get

$$p \times 4\pi R^2 \times dR = 16\pi R dRS$$

$$\boxed{p = \frac{4S}{R}}$$

Pressure difference inside a soap bubble,

$$p_i - p_0 = \frac{4S}{R}$$

Excess pressure inside an air bubble in a liquid is similar to a liquid drop in air, it has only one free spherical surface. Hence, excess pressure is given by

$$p = \frac{2S}{R}$$

Note

- When an air bubble of radius R lies at a depth h below the free surface of a liquid of density ρ and surface tension S, the excess pressure inside the bubble will be

$$p = \frac{2S}{R} + h\rho g$$

- When an ice-skater slides over the surface of smooth ice, some ice melts due to the pressure exerted by the sharp metal edges of the skates. The tiny water droplets act as rigid ball bearings and help the skaters to run along smoothly.

EXAMPLE |1| Excess Pressure in Hemispherical Dubble

The lower end of a capillary tube of diameter 2 mm is dipped 8 cm below the surface of water in a beaker. What is the pressure required in the tube in order to blow a hemispherical bubble at its end in water? The surface tension of water at temperature of the experiments is 7.30×10^{-2} Nm^{-1}, 1 atm $= 1.01 \times 10^5$ Pa, density of water $= 1000$ kg/m^3, $g = 9.80$ ms^{-2}. Also, calculate the excess pressure. [NCERT]

Sol. Given, $h = 0.08$ m, $d = 1000$ kg/m^3, $g = 9.80$ m/s^2 and we know that $p_0 = p_a + hdg$

p_0 is outside pressure

$$p_0 = 1.01 \times 10^5 \text{ Pa} + 0.08 \text{ m} \times 1000 \text{ kg/m}^3 \times 9.80 \text{ m/s}^2$$

$$p_0 = 1.01784 \times 10^5 \text{ Pa}$$

Calculate inside pressure required in tube in order to blow a hemispherical bubble

$$p_i = p_0 + \frac{2S}{r}$$

where $S = 7.30 \times 10^{-2}$ Pa-m

$$p_i = 1.01784 \times 10^5 + \frac{2 \times 7.3 \times 10^{-2}}{10^{-3}}$$

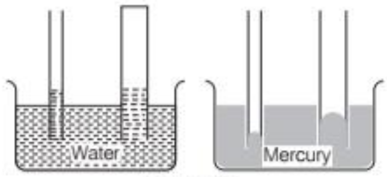
$$= (1.01784 + 0.00146) \times 10^5 \text{ Pa}$$

$$\Rightarrow \quad p_i = 1.02 \times 10^5 \text{ Pa}$$

where, the radius of the bubble is taken to be equal to the radius of capillary tube. Since, the bubble is hemispherical. The excess pressure in the bubble is 146 Pa.

CAPILLARITY

The term capilla means hair which is Latin word. A tube of very fine (hair-like) bore is called a **capillary tube**.



Capillarity

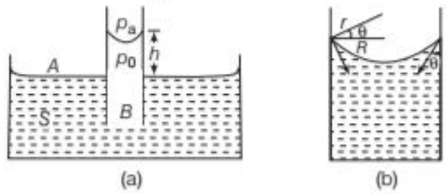
If a capillary tube of glass is dipped in liquid like water, the liquid rises in the tube, but when the capillary tube is dipped in a liquid like mercury, the level of liquid falls in the tube. This phenomenon of rise or fall of a liquid in the capillary is called **capillarity**.

Some examples are

1. We use towels for drying our skin.
2. In trees sap rises due to vessels. It is similar to capillary action.

Capillary Rise

One application of the pressure difference across a curved surface is the water rises up in a narrow tube (capillary) in spite of gravity. Consider a capillary of radius R is inserted into a vessel containing water.



Capillary rise

The surface of water in the capillary becomes concave. It means that there must be a pressure difference between the two sides of the meniscus.

So, $\quad (p_a - p_0) = (2S/r) = 2S/(R \sec\theta)$

$$= (2S/R)\cos\theta \quad \text{...(i)}$$

where, r = radius of curvature of the concave meniscus.

Now, consider two points A and B. According to Pascal's law, they must be at the same pressure,

$$p_0 + h\rho g = p_A = p_B = p_a$$

So, $\quad p_a = p_0 + h\rho g$

$$p_a - p_0 = \rho g h \quad \text{...(ii)}$$

(p_a = atmospheric pressure)

From Eqs. (i) and (ii), we get

$$\rho g h=\frac{2S}{R}\cos\theta$$

Height of rise of liquid in capillary, $h=\frac{2S\cos\theta}{\rho R g}$

This is the formula for the rise of liquid in a capillary. The liquids which met the glass surfaces, e.g. water, rises in the capillary and the liquids which do not wet the glass surface falls in the capillary.

Some common examples of capillarity are

(i) Blotting paper absorbs ink due to capillarity.
(ii) A towel soaks water on account of capillarity motion.
(iii) Oil rises through the wicks due to capillarity.

EXAMPLE |2| Water Rise in Capillary

A capillary of radius 0.05 cm is immersed in water. Find the value of rise of water in capillary if value for the surface tension is 0.073 N/m and angle of contact is 0°.

Sol. Given, $S=0.073$ N/m,

$$R=0.05\,\text{cm}=5\times10^{-4}\,\text{m}$$
$$\theta=0^\circ,\ h=?$$

From the formula, $h=\frac{2S\cos\theta}{\rho g R}$

$$=\frac{2\times0.073\times\cos0^\circ}{10^3\times9.8\times5\times10^{-4}}=0.02979\,\text{m}$$

EXAMPLE |3| Different Liquid in Capillary Tube

In a glass capillary tube, water rises upto a height of 10.0cm while mercury fall down by 5.0cm in the same capillary. If the angles of contact for mercury glass is 60° and water glass is 0°, then find the ratio of surface tension of mercury and water.

Sol. For water, $h_1=10.0\,\text{cm}=0.1\,\text{m}$

$$\rho_1=10^3\,\text{kg/m}^3,\ \theta=0^\circ$$

For mercury, $h_2=5.0\,\text{cm}=0.05\,\text{m}$

$$\rho_2=13.6\times10^3\,\text{kg/m}^3,\ \theta=60^\circ$$

Suppose S_1 and S_2 are the surface tensions for water and mercury, respectively, then

$$S_1=\frac{h_1R\rho_1 g}{2\cos\theta_1}\ \text{and}\ S_2=\frac{h_2R\rho_2 g}{2\cos\theta_2}$$

The ratio of surface tension of mercury and water,

$$\frac{S_2}{S_1}=\frac{h_2R\rho_2 g}{2\cos\theta_2}\times\frac{2\cos\theta_1}{h_1R\rho_1 g}$$

$$\frac{S_2}{S_1}=\frac{h_2\rho_2\cos\theta_1}{h_1\rho_1\cos\theta_2}$$

$$=\frac{0.05\times13.6\times10^3\times\cos0^\circ}{0.1\times1000\times\cos60^\circ}=13.6:1$$

TOPIC PRACTICE 2

OBJECTIVE Type Questions

1. Surface tension is due to

(a) frictional forces between molecules
(b) cohesive forces between molecules
(c) adhesive forces between molecules
(d) Both (b) and (c)

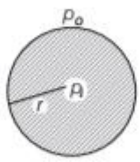
Sol. (d) Both cohesive and adhesive forces result in surface tension.

2. The value of surface tension of water is minimum at

(a) 4° C (b) 25° C
(c) 50° C (d) 75° C

Sol. (d) Value of surface tension decreases with increase in temperature. So, it is minimum at 75°C.

3. In figure, pressure inside a spherical drop is more than pressure outside. (S = surface tension and r = radius of bubble)



The extra surface energy if radius of bubble is increased by Δr is

(a) $4\pi r\,\Delta r\,S$ (b) $8\pi r\,\Delta r\,S$
(c) $2\pi r\,\Delta r\,S$ (d) $10\pi r\,\Delta r\,S$

Sol. (b) Suppose a spherical drop of radius r is in equilibrium. If its radius increases by Δr. The extra surface energy is

$$|4\pi(r+\Delta r)^2-4\pi r^2|S=8\pi r\,\Delta r\,S$$

4. Radius of a soap bubble is increased from R to $2R$. Work done in this process in terms of surface tension is

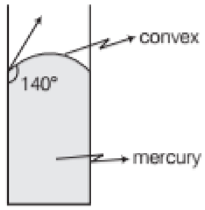
(a) $24\,\pi R^2S$ (b) $48\,\pi R^2S$
(c) $12\pi R^2S$ (d) $36\pi R^2S$

Sol. (a) $W=8\pi T(R_2^2-R_1^2)=8\pi S[(2R)^2-(R^2)]=24\,\pi R^2S$

5. The angle of contact at the interface of water-glass is 0°, ethyl alcohol-glass is 0°, mercury-glass is 140° and methyliodide-glass is 30°. A glass capillary is put in a trough containing one of these four liquids. It is observed that the meniscus is convex. The liquid in the trough is **[NCERT Exemplar]**

(a) water (b) ethylalcohol
(c) mercury (d) methyliodide

Sol. (c) According to the question, the observed meniscus is of convex figure shape. Which is only possible when angle of contact is obtuse. Hence, the combination will be of mercury-glass (140°)



VERY SHORT ANSWER Type Questions

6. If a wet piece of wood burns, then water droplets appear on the other end, why?

Sol. When a piece of the wood burns, then steam formed and water appear in the form of drops due to surface tension on the other end.

7. Why, surface tension of all lubricating oils and paint is kept low?

Sol. So, they can spread over large area easily.

8. It becomes easier to spray the water in which some soap is dissolved, explain it.

Sol. When some soap is dissolved in water, the surface tension of water decreases. Thus, the less energy required to spray water.

9. Find the work done in increasing the radius of a soap bubble from 4 cm to 6 cm. The value of surface tension for the soap solution is 30 dyne cm^{-1}.

Sol. Given, $r_1 = 4$ cm, $r_2 = 6$ cm, $S = 30$ dyne cm^{-1}

So, change in surface area $= 2 \times 4\pi (6^2 - 4^2)$

[Soap solution has two surfaces]

$= 8\pi\ 20 = 160\ \pi\ cm^2$

Work done $= S \times$ change in surface area

$= 30 \times 160 \times 3.142$

$= 15081.6$ erg

10. Find the work done required to make a soap bubble of radius 0.02 m. Given surface tension of soap 0.03 N/m.

Sol. Given, $S = 0.03$ N/m

Work done = surface area × surface tension

$= 2 \times 4\pi r^2 \times S$

$= 2 \times 4 \times 3.14 \times (0.02)^2 \times 0.03$

$= 3 \times 10^{-4}$ J

11. Why soap bubble bursts after some time?

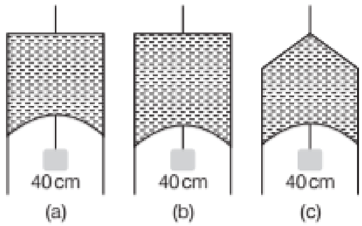
Sol. Soap bubble bursts after some time because the pressure inside it become more than the outside pressure.

12. How a raincoat become waterproof?

Sol. If the angle of contact between water and the material of the raincoat is obtuse, then rainy water does not wet the raincoat. Thus, the raincoat becomes waterproof.

SHORT ANSWER Type Questions

13. Fig.(a) shows a thin liquid film supporting a small weight $= 4.5 \times 10^{-2}$N. What is the weight supported by a film of the same liquid at the same temperature in Figs. (b) and (c)? Explain your answer physically. **[NCERT]**



Sol. As liquid is same, temperature is same and the length of the film supporting the weight is also same, therefore, in Figs. (b) and (c), the film will support same weight, i.e. 4.5×10^{-2}N.

14. The surface tension and vapour pressure of water at 20°C is 7.28×10^{-2}N/m and 2.33×10^3Pa, respectively. What is the radius of the smallest spherical water droplet which can form without evaporating at 20°C? **[NCERT]**

Sol. Given, surface tension of water, $S = 7.28 \times 10^{-2}$ N/m

Vapour pressure, $p = 233 \times 10^3$ Pa

The drop will evaporate, if the water pressure is greater than the vapour pressure.

Let a water droplet of radius R can be formed without evaporating.

∴ Vapour pressure = Excess pressure in drop

$$\therefore \quad p = \frac{2S}{R}$$

$$\text{or} \quad R = \frac{2S}{p} = \frac{2 \times 7.28 \times 10^{-2}}{233 \times 10^3} = 6.25 \times 10^{-5} \text{ m}$$

15. A liquid drop breaks into 27 small drops. If surface tension of the liquid is S, then find the energy released.

Sol. Let the radius of larger drop = R

and radius of each small drop = r

Volume of 27 small drops = Volume of the large drop

$$= 27 \times \frac{4}{3} \times \pi r^3 = \frac{4}{3}\pi R^3$$

So, $r = R/3$

Surface area of large drop $= 4\pi R^2$

Surface area of 27 small drops $= 27 \times 4\pi r^2$

$$= 27 \times 4\pi \left(\frac{R}{3}\right)^2 = 12\pi R^2$$

$\therefore$ Increase in surface area $= 12\pi R^2 - 4\pi R^2 = 8\pi R^2$

Increase in energy = Increase in surface area × Surface tension

$$= 8\pi R^2 \times S$$

16. Two soap bubbles of radii 6 cm and 8 cm coalesce to form a single bubble. Find the radius of the new bubble.

Sol. Surface energy of first soap bubble

= Surface tension × surface area

$= 2 \times 4\pi R_1^2 S = 8\pi R_1^2 S$

Surface energy of second soap bubble $= 8\pi R_2^2 S$

Let the radius of the new soap bubble is R. So, the surface energy of new bubble $= 8\pi R^2 S$

By the law of conservation of energy,

$$8\pi R^2 S = 8\pi R_1^2 S + 8\pi R_2^2 S$$

$$R^2 = R_1^2 + R_2^2 = 36 + 64$$

$$\therefore \quad R^2 = 100 \text{ cm}^2 \Rightarrow R = 10 \text{ cm}$$

17. A liquid drop of radius 4 mm breaks into 1000 identical drops. Find the change in surface energy. $S = 0.07 \text{ Nm}^{-1}$.

Sol. Volume of 1000 small drops = Volume of a large drop

$$1000 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

$$r = \frac{R}{10}$$

Surface area of large drop $= 4\pi R^2$

Surface area of 1000 drop $= 4\pi \times 1000\, r^2 = 40\pi R^2$

$\therefore$ Increase in surface area $= (40-4)\pi R^2 = 36\,\pi R^2$

The increase in surface energy

= Surface tension × increase in surface area

$= 36\pi R^2 \times 0.07 = 36 \times 3.14 \times (4 \times 10^{-3})^2 \times 0.07$

$= 1.26 \times 10^{-4}$ J

18. The sap in trees, which consists mainly of water in summer, rises in a system of capillaries of radius $r = 2.5 \times 10^{-5}$ m. The surface tension of sap is $S = 7.28 \times 10^{-2}$ N/m and angle of contact is 0°. Does surface tension alone account for the supply of water to the top of all trees? **[NCERT]**

Sol. Given, radius $(r) = 2.5 \times 10^{-5}$ m

Surface tension $(S) = 7.28 \times 10^{-2}$ N/m

Angle of contact $(\theta) = 0°$,

density of water $(\rho) = 10^3$ kg/m³

The maximum height which sap can rise in trees through capillarity action is given by

$$h = \frac{2S \cos\theta}{r\rho g}$$

$$h = \frac{2 \times 7.28 \times 10^{-2} \times \cos 0°}{2.5 \times 10^{-5} \times 1 \times 10^3 \times 9.8} = 0.59 \text{ m}$$

But, the height of many trees are more than 0.59 m, therefore, the rise of sap in all trees is not possible through capillarity action alone.

LONG ANSWER Type I Questions

19. A U-shaped wire is dipped in a soap solution and removed. The thin soap film formed between the wire and a light slider supports a weight of 1.5×10^{-2} N (which includes the small weight of the slider). The length of the slider is 30 cm. What is the surface tension of the film? **[NCERT]**

Sol. Length of the slider $(l) = 30$ cm

As a soap film has two free surfaces, therefore, total length of the film to be supported

$$l' = 2l = 2 \times 30 = 60 \text{ cm} = 0.60 \text{ m}$$

Let S be the surface tension of the soap solution.

Total force on the slider due to surface tension,

$$F = S \times 2l$$

$$F = S \times 0.60 \text{ N} \quad \text{...(i)}$$

Weight (w) supported by the slider $= 1.5 \times 10^{-2}$ N

In equilibrium,

Force on the slider due to surface tension = weight supported by the slider

$$\therefore \quad F = w$$

$$S \times 0.60 = 1.5 \times 10^{-2}$$

or

$$S = \frac{1.5 \times 10^{-2}}{0.60} = 2.5 \times 10^{-2} \text{ N/m}$$

20. What is the pressure inside the drop of mercury of radius 3.00 mm at room temperature? Surface tension of mercury at that temperature (20°C) is 4.65×10^{-1} N/m. The atmospheric pressure is 1.01×10^5 Pa. Also, give the excess pressure inside the drop.

Excess pressure inside a liquid drop is given by $\Delta p = \frac{2S}{R}$, where S = surface tension of the liquid, R = radius of the drop. **[NCERT]**

Sol. Given, radius of drop $(R) = 3.00$ mm $= 3 \times 10^{-3}$ m

Surface tension of mercury $(S) = 4.65 \times 10^{-1}$ N/m

Atmospheric pressure $(p_0) = 1.01 \times 10^5$ Pa

Pressure inside the drop = Atmospheric pressure + Excess pressure inside the liquid drop $= p_0 + \frac{2S}{R}$

$$= 1.01 \times 10^5 + \frac{2 \times 4.65 \times 10^{-1}}{3 \times 10^{-3}}$$
$$= 1.01 \times 10^5 + 3.10 \times 10^2$$
$$= 1.01 \times 10^5 + 0.00310 \times 10^5$$
$$= 1.01310 \times 10^5 \text{ Pa}$$

Excess pressure inside the drop

$$(\Delta p) = \frac{2S}{R} = \frac{2 \times 4.65 \times 10^{-1}}{3 \times 10^{-3}}$$
$$= 3.10 \times 10^2 = 310 \text{ Pa}$$

21. Two mercury droplets of radii 0.1 cm and 0.2 cm collapse into one single drop. What amount of energy is released? The surface tension of mercury $S = 435.5 \times 10^{-3}$ N/m.

When two or more droplets collapse to form a bigger drop, then its surface area decreases and energy is released equal to $S\Delta A$. **[NCERT]**

Sol. Radii of mercury droplets, $r_1 = 0.1$ cm $= 1 \times 10^{-3}$ m

$r_2 = 0.2$ cm $= 2 \times 10^{-3}$ m

Surface tension $(S) = 435.5 \times 10^{-3}$ N/m

Let the radius of the big drop formed by collapsing be R.

$\therefore$ Volume of big drop = Volume of small droplets

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3$$

or
$$R^3 = r_1^3 + r_2^3$$
$$= (0.1)^3 + (0.2)^3$$
$$= 0.001 + 0.008 = 0.009$$

or $R = 0.21$ cm $= 2.1 \times 10^{-3}$ m

$\therefore$ Change in surface area

$$\Delta A = 4\pi R^2 - (4\pi r_1^2 + 4\pi r_2^2)$$
$$= 4\pi [R^2 - (r_1^2 + r_2^2)]$$

$\therefore$ Energy released $= S \cdot \Delta A$

$$= S \times 4\pi [R^2 - (r_1^2 + r_2^2)]$$
$$= 435.5 \times 10^{-3} \times 4 \times 3.14 [(2.1 \times 10^{-3})^2 - (1 \times 10^{-6} + 4 \times 10^{-6})]$$
$$= 435.5 \times 4 \times 3.14 [4.41 - 5] \times 10^{-6} \times 10^{-3}$$
$$= -32.27 \times 10^{-7} = -3.22 \times 10^{-6} \text{ J}$$

(Negative sign shows absorption)

Therefore, 3.22×10^{-6} J energy will be absorbed.

22. If a drop of liquid breaks into smaller droplets, it results in lowering of temperature of the droplets. Let a drop of radius R, break into N small droplets each of radius r. Estimate the lowering in temperature. **[NCERT]**

Sol. When a big drop of radius R, break into N droplets each of radius r, the volume remains constant.

$\therefore$ Volume of big drop = $N \times$ Volume of small drop

$$\frac{4}{3}\pi R^3 = N \times \frac{4}{3}\pi r^3$$

or
$$R^3 = Nr^3 \quad \text{or} \quad N = \frac{R^3}{r^3}$$

Now, change in surface area $= 4\pi R^2 - N4\pi r^2$
$$= 4\pi (R^2 - Nr^2)$$

Energy released $= S \times \Delta A = S \times 4\pi (R^2 - Nr^2)$

Due to releasing of this energy, the temperature is lowered.

If ρ is the density and s is specific heat of liquid and its temperature is lowered by $\Delta\theta$, then

Energy released $= ms\Delta\theta$

$$S \times 4\pi (R^2 - Nr^2) = \left(\frac{4}{3}\pi \times R^3 \times \rho\right) s\Delta\theta$$

$$\Delta\theta = \frac{S \times 4\pi (R^2 - Nr^2)}{\frac{4}{3}\pi R^3 \rho \times s} = \frac{3S}{\rho s}\left[\frac{R^2}{R^3} - \frac{Nr^2}{R^3}\right]$$

$$= \frac{3S}{\rho s}\left[\frac{1}{R} - \frac{(R^3/r^3) \times r^2}{R^3}\right] \qquad \left[\because N = \frac{R^3}{r^3}\right]$$

$$\Delta\theta = \frac{3S}{\rho s}\left[\frac{1}{R} - \frac{1}{r}\right]$$

LONG ANSWER Type II Questions

23. If a number of little droplets of water, each of radius r, coalesce to form a single drop of radius R, and the energy released is converted into kinetic energy then find out the velocity acquired by the bigger drop.

Sol. Let n be the number of little droplets which coalesce to form single drop. Then,

Volume of n little droplets = Volume of single drop

or $\quad n \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$ or $nr^3 = R^3$

Decrease in surface area

$$= n \times 4\pi r^2 - 4\pi R^2$$
$$= 4\pi [nr^2 - R^2] = 4\pi \left[\frac{nr^3}{r} - R^2\right]$$
$$= 4\pi \left[\frac{R^3}{r} - R^2\right] = 4\pi R^3 \left[\frac{1}{r} - \frac{1}{R}\right]$$

The energy released, $[\because nr^3 = R^3]$

E = Surface tension × decrease in surface area

$$= 4\pi SR^3\left[\frac{1}{r}-\frac{1}{R}\right]$$

The mass of bigger drop,

$$M = \frac{4}{3}\pi R^3 \times 1 = \frac{4}{3}\pi R^3$$

$$\therefore \quad E = \frac{4}{3}\pi SR^3 . 3\left[\frac{1}{r}-\frac{1}{R}\right]$$

$$= 3SM\left[\frac{1}{r}-\frac{1}{R}\right] \qquad \left[\because M = \frac{4}{3}\pi R^3\right]$$

$\because$ KE of bigger drop = Energy released

$$\frac{1}{2}MV^2 = 3SM\left[\frac{1}{r}-\frac{1}{R}\right]$$

$$\therefore \quad V = \sqrt{6S\left(\frac{R-r}{Rr}\right)}$$

24. What is the excess pressure inside a bubble of soap solution of radius 5.00 mm, given that the surface tension of soap solution at the temperature (20°C) is 2.50×10^{-2} N/m? If an air bubble of the same dimension were formed at a depth of 40.0 cm inside a container containing the soap solution (of relative density 1.20), what would be the pressure inside the bubble? (1 atm pressure is 1.01×10^5 Pa.)

The excess pressure inside a soap bubble is given by $\Delta p = \frac{4S}{R}$. where S = surface tension of the soap solution, R = radius of the bubble.

[NCERT]

Sol. Given, surface tension of soap solution (S)

$$= 2.5 \times 10^{-2}\,\text{N/m}$$

Density of soap solution $(\rho) = 1.2 \times 10^3\,\text{kg/m}^3$

Radius of soap bubble $(r) = 5.00$ mm $= 5.0 \times 10^{-3}$ m

Radius of air bubble $(R) = 5 \times 10^{-3}$ m

Atmospheric pressure $(p_0) = 1.01 \times 10^5$ Pa

Excess pressure inside the soap bubble

$$= \frac{4S}{r} = \frac{4 \times 2.5 \times 10^{-2}}{5.0 \times 10^{-3}}$$

$$= 20\ \text{Pa}$$

Excess pressure inside the air bubble

$$= \frac{2S}{R} = \frac{2 \times 2.5 \times 10^{-2}}{5.0 \times 10^{-3}} = 10\ \text{Pa}$$

$\therefore$ Pressure inside the air bubble = Atmospheric pressure + Pressure due to 40 cm of soap solution column + Excess pressure inside the bubble

$$= (1.01 \times 10^5) + (0.40 \times 1.2 \times 10^3 \times 9.8) + 10$$

$$= (1.01 \times 10^5) + 4.704 \times 10^3 + 10$$

$$= 1.01 \times 10^5 + 0.04704 \times 10^5 + 0.00010 \times 10^5$$

$$= 1.05714 \times 10^5\,\text{Pa} = 1.06 \times 10^5\,\text{Pa}$$

25. Explain why?

(i) The angle of contact of mercury with glass is obtuse, while that of water with glass is acute.

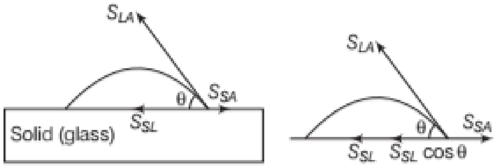
(ii) Water on a clean glass surface tends to spread out while mercury on the same surface tends to form drops. (Put differently, water wets the glass while mercury does not.)

(iii) Surface tension of a liquid is independent of the area of the surface.

(iv) Water with detergents dissolved in it should have small angles of contact.

(v) A drop of liquid under no external forces is always spherical in shape. **[NCERT]**

Sol. (i) When a small quantity of a liquid is poured on a solid, three types of interfaces namely liquid-air, solid-air and solid-liquid are occured. The surface tension corresponding to these three interfaces are S_{LA}, S_{SA} and S_{SL}, respectively. If θ is the angle of contact between solid and liquid then,



$$S_{SL} + S_{LA}\cos\theta = S_{SA}$$

$$\Rightarrow \quad S_{LA}\cos\theta = S_{SA} - S_{SL}$$

$$\cos\theta = \frac{S_{SA} - S_{SL}}{S_{LA}} \qquad \text{...(i)}$$

For mercury and glass $S_{SA} < S_{SL}$, therefore, from Eq. (i) $\cos\theta$ is negative and therefore, $\theta > 90°$, i.e. obtuse.

For water and glass $S_{SA} > S_{SL}$, therefore, from Eq. (i) $\cos\theta$ is positive and therefore, $\theta < 90°$, i.e. acute.

(ii) For equilibrium of a liquid drop on a solid surface

$$S_{SL} + S_{LA}\cos\theta = S_{SA}$$

In case of mercury and glass, $S_{SL} > S_{SA}$, therefore, for equilibrium $\cos\theta$ should be negative, i.e. θ should be obtuse. In order to achieve this obtuse value of angle of contact, the mercury tends to form a drop. In case of water and glass, $S_{SA} > S_{SL}$, therefore, for equilibrium $\cos\theta$ should be positive, i.e. θ should be acute.

In order to achieve this value of angle of contact, the water tends to spread.

(iii) Surface tension of a liquid is defined as the force acting per unit length on an imaginary line drawn tangentially to the liquid surface at rest. Therefore, surface tension is independent of the area of the liquid surface.

(iv) The rise of liquid in a capillary tube is given by

$$h = \frac{2S \cos \theta}{r \rho g},$$

i.e. $h \propto \cos \theta$

The cloth has narrow spaces in the form of fine capillaries. If angle of contact θ is small, then the value of $\cos \theta$ will be large and hence, detergent will rise more in fine capillaries in the cloth. Now, the detergent solution will penetrate more in cloth and remove stains and dust from the cloth.

(v) In the absence of external forces, the size of a liquid drop is decided only by the force due to surface tension. Due to surface tension, a liquid drop tends to acquire minimum surface area. As surface area is minimum for a sphere for a given volume of liquid. Therefore, under no external force a liquid is always spherical in shape.

26. Mercury has an angle of contact equal to 140° with sodalime glass. A narrow tube of radius 1.00 mm made of this glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside? Surface tension of mercury at the temperature of the experiment is 0.465 N/m. Density of mercury $= 13.6 \times 10^3 \text{kg/m}^3$. **[NCERT]**

Sol. Given, angle of contact $(\theta) = 140°$

Radius of tube $(r) = 1 \text{ mm} = 10^{-3} \text{m}$

Surface tension $(S) = 0.465 \text{ N/m}$

Density of mercury $(\rho) = 13.6 \times 10^3 \text{kg/m}^3$

Height of liquid rise or fall due to surface tension (h)

$$= \frac{2S \cos \theta}{r \rho g}$$

$$= \frac{2 \times 0.465 \times \cos 140°}{1 \times 10^{-3} \times 13.6 \times 10^3 \times 9.8}$$

$$= \frac{2 \times 0.465 \times (-0.7660)}{10^{-3} \times 13.6 \times 10^3 \times 9.8}$$

$$= -5.34 \times 10^{-3} \text{m}$$

$$= -5.34 \text{ mm}$$

Hence, the mercury level will depressed by 5.34 mm.

27. Two narrow bores of diameter 3.0 mm and 6.0 mm are joined together to form a U-tube open at both ends. If the U-tube contains water, what is the difference in its levels in the two limbs of the tube? Surface tension of water at the temperature of the experiment is $7.3 \times 10^{-2} \text{N/m}$. Take the angle of contact to be zero and density of water to be $1.0 \times 10^3 \text{kg/m}^3$. $(g = 9.8 \text{ m/s}^2)$ **[NCERT]**

Sol. Given, surface tension of water $(S) = 7.3 \times 10^{-2} \text{N/m}$

Density of water $(\rho) = 1.0 \times 10^3 \text{kg/m}^3$

Acceleration due to gravity $(g) = 9.8 \text{ m/s}^2$

Angle of contact $(\theta) = 0°$

Diameter of one side, $2r_1 = 3.0 \text{ mm}$

$\therefore \quad r_1 = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{m}$

Diameter of other side, $2r_2 = 6.0 \text{ mm}$

$r_2 = 3.0 \text{ mm} = 3.0 \times 10^{-3} \text{m}$

Height of water column rises in first and second tubes

$$h_1 = \frac{2S \cos \theta}{r_1 \rho g} \Rightarrow h_2 = \frac{2S \cos \theta}{r_2 \rho g}$$

$\therefore$ Difference in levels of water rises in both tubes

$$\Delta h = h_1 - h_2 = \frac{2S \cos \theta}{\rho g} \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

$$= \frac{2 \times 7.3 \times 10^{-2} \times \cos 0°}{1.0 \times 10^3 \times 9.8} \left[\frac{1}{1.5 \times 10^{-3}} - \frac{1}{3.0 \times 10^{-3}} \right]$$

$$= \frac{14.6}{9.8} \times 10^{-2} \left[\frac{2-1}{3} \right]$$

$$= \frac{14.6}{9.8 \times 3} \times 10^{-2}$$

$$= 0.497 \times 10^{-2} \text{m}$$

$$= 4.97 \times 10^{-3} \text{m}$$

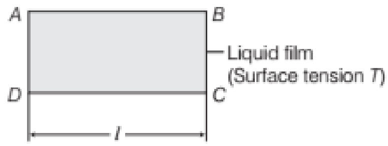
$$= 4.9 \text{ mm}$$

ASSESS YOUR TOPICAL UNDERSTANDING

OBJECTIVE Type Questions

1. The surface tension of a liquid at its boiling point
(a) becomes zero
(b) becomes infinity
(c) is equal to the value at room temperature
(d) is half to the value at the room temperature

2. A liquid film is formed over a frame $ABCD$ as shown in figure. Wire CD can slide without friction. Maximum value of mass that can be hanged from CD without breaking the liquid film is

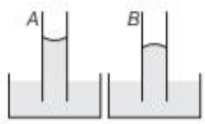


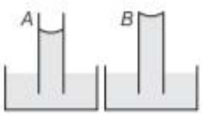
(a) $\frac{Tl}{g}$ (b) $\frac{2Tl}{g}$
(c) $\frac{g}{2Tl}$ (d) $T \times l$

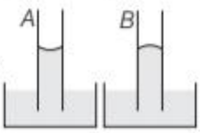
3. The force required to separate two glass plates of $10^{-2}\,m^2$ with a film of water 0.05 mm thick between them, is (surface tension of water is $70 \times 10^{-3}\,Nm^{-1}$)
(a) 28 N (b) 14 N (c) 50 N (d) 38 N

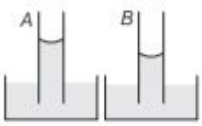
4. Why are drops and bubbles spherical?
(a) Surface with minimum energy
(b) Surface with maximum energy
(c) High pressure
(d) Low pressure

5. A capillary tube A is dipped in water. Another identical tube B is dipped in soap-water solution. Which of the following shows the relative nature of the liquid columns in the two tubes?

(a)


(b)


(c)


(d)


Answer

1. (a)	**2.** (b)	**3.** (a)	**4.** (a)	**5.** (d)

VERY SHORT ANSWER Type Questions

6. Find the pressure inside an air bubble of radius 5 cm which is 5 cm below the surface of water. Surface tension of water is $7 \times 10^{-2}\ Nm^{-1}$.

7. Find force to pull a circular disc of radius 2 cm from surface of water. Surface tension of water = 0.07 N/m.

8. Why two boats which are moving close to each other, pushed towards each other?

SHORT ANSWER Type Questions

9. The excess pressure inside a soap bubble of radius 4 cm is balanced by the weight of 20 mg, then find the value of surface tension.

10. Water rises upto height h in a capillary tube of certain diameter. This capillary tube is replaced by similar tube of half the diameter. Find out the height of water in that capillary.

LONG ANSWER Type I Questions

11. A frame made of metallic wire enclosing a surface area A is covered with a soap film. If the area of the frame of metallic wire is reduced by 50%, then what will be the change in energy of the soap.

12. A liquid rises in a capillary such that the surface tension balances its weight of 5×10^{-3} N of liquid, surface tension of liquid is $5 \times 10^{-2}\ Nm^{-1}$. Find the radius of the capillary.

LONG ANSWER Type II Questions

13. Two capillary tubes of same diameter are put vertically one each in two liquids whose relative densities are 0.8 and 0.6 and surface tensions are 60 and 50 dyne/cm, respectively. What is the ratio of heights of liquids in the two tubes?

14. The limbs of a manometer consist of uniform capillary tubes of radii 1.44×10^{-3} m and 7.2×10^{-4} m. Find out the correct pressure difference if the level of the liquid (density $10^3\ kgm^{-3}$ surface tension $72 \times 10^{-3}\ Nm^{-1}$) in the narrower tube stands 0.2 m above that in the broader tube.

|TOPIC 3|
Hydrodynamics

We have studied the fluids at rest or hydrostatics. Now, we will learn the branch of Physics which deals with the study of fluids in motion called **fluid dynamics** or **hydrodynamics**.

STREAMLINE, LAMINAR AND TURBULENT FLOW

Streamline Flow

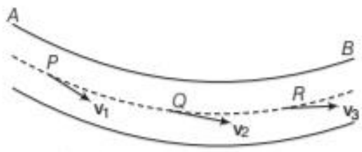
Streamline flow of a liquid is that flow in which each particle of the liquid passing through a point travels along the same path and with the same velocity as the preceding particle passing through the same point. It is also defined as a curve whose tangent at any point is in the direction of the fluid velocity at that point.

Trajectory of a fluid particle

Properties of Streamline

(i) In streamline flow, no two streamlines can cross each other. If they do so, the particles of the liquid at the point of intersection will have two different directions for their flow, which will destroy the steady nature of the liquid flow.

(ii) The greater is the crowding of streamline at a place greater is the velocity of the liquid particles at that place and *vice-versa*.



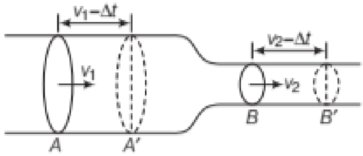
A region of streamline flow

Equation of Continuity

It states that, **during the streamline flow of the non-viscous and incompressible fluid through a pipe of varying cross-section, the product of area of cross-section and the normal fluid velocity (av) remains constant throughout the flow.**

Consider a non-viscous and incompressible liquid flowing through a tube AB of varying cross-section.

Let a_1 be the area of cross-section, v_1 at section A and the values of corresponding quantities at section B be a_2 and v_2, respectively. The density of flowing fluid is ρ.



Fluid flow through pipe AB

As the fluid is incompressible, is constant

As, mass = volume × density

= Area of cross-section × length × density

∴ Mass of fluid that entering through section A in time Δt,

$$m_1 = a_1 v_1 \Delta t \rho_1$$

Mass of fluid that leaving through section B in time Δt,

$$m_2 = a_2 v_2 \Delta t \rho_2$$

According to conservation of mass, we get

$$m_1 = m_2$$

or $$a_1 v_1 \Delta t \rho = a_2 v_2 \Delta t \rho$$

ρ, is constant

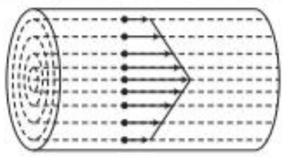
Hence, $$a_1 v_1 = a_2 v_2$$

or $$\boxed{av = \text{constant}}$$

This is known as **equation of continuity**.

Laminar Flow

If the liquid flows over a horizontal surface in the form of layers of different velocities, then the flow of liquid is called **laminar flow**.


Laminar flow of liquid

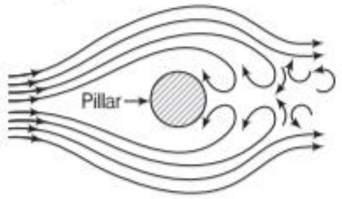
In laminar flow, the particle of one layer do not enter into another layer. In general, laminar flow is a streamline flow as shown in figure.

Turbulent Flow

In rivers and canals, where speed of water is quite high or the boundary surfaces cause abrupt changes in velocity of the flow, then the flow becomes irregular.

Such flow of liquid is known as **turbulent flow**.

Thus, the flow of fluid in which velocity of all particles crossing a given point is not same and the motion of the fluid becomes irregular or disordered is called **turbulent flow** as shown in figure.



Turbulent flow of liquid

Ideal Fluid

The motion of real fluids is very complicated. To understand fluid dynamics in a simpler manner, we assume that the fluid is ideal. An ideal fluid is one which is non-viscous, incompressible, and its flow is steady and irrotational.

BERNOULLI'S THEOREM

Bernoulli's principle is based on the law of conservation of energy and applied to ideal fluids. It states that **the sum of pressure energy per unit volume, kinetic energy per unit volume and potential energy per unit volume of an incompressible, non-viscous fluid in a streamlined irrotational flow remains constant at every cross-section throughout the liquid flow.**

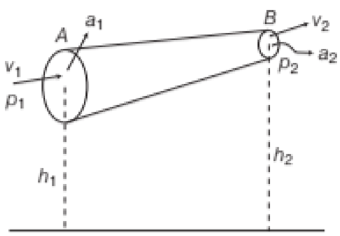
Mathematically, it can be expressed as

$$\boxed{p + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}}$$

where, p represents for pressure energy per unit volume $\frac{1}{2}\rho v^2$ for kinetic energy per unit volume and ρgh for potential energy per unit volume and ρ is density of flowing fluid (ideal).

The Swiss physicist **Daniel Bernoulli** developed this relationship in 1738.

Proof Consider an ideal fluid having streamline flow through a pipe of varying area of cross-section as shown in figure.



The flow of an ideal fluid in a pipe varying cross-section

Let p_1, a_1, h_1, v_1 and p_2, a_2, h_2, v_2 be the pressure, area of cross-section, height and velocity of flow at points A and B, respectively. Force acting on fluid at point $A = p_1 a_1$

Distance travelled by fluid in one second at point A

$$= v_1 \times 1 = v_1$$

Work done per second on the fluid at point A

$$= \text{Force} \times \text{distance travelled by fluid in one second.}$$

$$W_1 = p_1 a_1 \times v_1$$

Similarly, work done per second by the fluid at point B,

$$W_2 = p_2 a_2 v_2$$

$\therefore$ Net work done on the fluid by pressure energy,

$$W = p_1 a_1 v_1 - p_2 a_2 v_2$$

But, $\quad a_1 v_1 = a_2 v_2 = \frac{m}{\rho}$ (i.e. equation of continuity)

$\therefore$ Net work done on the fluid by the pressure energy,

$$W = \left(\frac{p_1 m}{\rho} - \frac{p_2 m}{\rho}\right)$$

Total work done by the pressure energy on the fluid increases the kinetic energy and potential energy of the fluid, when it flows from A to B.

$\therefore$ Increase in potential energy of fluid

$$= \text{KE at } B - \text{KE at } A$$

$$= \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 \qquad [\because a_1 > a_2 \therefore v_2 > v_1]$$

Similarly, total increase in potential energy $= mgh_2 - mgh_1$

According to work energy theorem, work done on the fluid is equal to change in the energy of fluid.

i.e. work done by the pressure energy = total increase in energy

$$\therefore \quad \frac{p_1 m}{\rho} - \frac{p_2 m}{\rho} = (mgh_2 - mgh_1) + \left(\frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2\right)$$

$$\left(\frac{p_1 - p_2}{\rho}\right) = (gh_2 - gh_1) + \frac{1}{2} v_2^2 - \frac{1}{2} v_1^2$$

or $$\frac{p_1}{\rho} + \frac{1}{2} v_1^2 + gh_1 = \frac{p_2}{\rho} + \frac{1}{2} v_2^2 + gh_2$$

Hence, $$\frac{p}{\rho} + gh + \frac{1}{2} v^2 = \text{constant}$$

$$\boxed{p + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}} \qquad \text{...(i)}$$

Dividing both sides of Eq. (i) by ρg, we get

$$\frac{p}{\rho g} + h + \frac{v^2}{2g} = \frac{\text{constant}}{\rho g} = \text{new constant} \qquad \text{...(ii)}$$

Here, $\frac{p}{\rho g}$ is called pressure head, h is called gravitational head and $\frac{v^2}{2g}$ is called velocity head.

Equation (ii) enable us to state Bernoulli's theorem in the streamline flow of an ideal liquid as the sum of pressure head, gravitational head and velocity head is always constant. If the fluid is flowing through a horizontal tube, two ends of the tube are at the same level. Therefore, there is no gravitational head (level difference), i.e. $h = 0$.

$$\frac{p}{\rho} + \frac{1}{2}v^2 = p + \frac{1}{2}\rho v^2 = \text{constant}$$

This shows, if **p increases, v decreases and *vice-versa*.** Thus, Bernoulli's theorem also states that in the streamline flow of an ideal liquid through a horizontal tube, the velocity increases where pressure decreases and *vice-versa*. This is also called **Bernoulli's principle**.

Bernoulli's Equation for the Fluid at Rest

When a fluid at rest, i.e. the velocity is zero everywhere, then the Bernoulli's equation becomes

$$p_1 + \rho g h_1 = p_2 + \rho g h_2$$

$$\boxed{p_1 - p_2 = \rho g (h_2 - h_1)}$$

Note

It must be noted that while applying the conservation of energy principle, then we assumed that there is no loss in energy due to friction. But in fact, when the fluid flow, some of the energy is lost due to friction. Generally, when the fluid flows in the different layers with different velocities. These layers exert frictional forces. This property of fluid is called **viscosity**.

Limitations of Bernoulli's Theorem

(i) Bernoulli's equation ideally applies to fluids with zero viscosity or non-viscous fluids.

(ii) The fluids must be incompressible, as the elastic energy of the fluid is also not taken into consideration.

(iii) Bernoulli's equation is applicable only to streamline flow of a fluid. It is not valid for non-steady or turbulent flow.

EXAMPLE |1| Water Flowing Through Two Pipes

Consider the two horizontal pipes of different diameters which are connected together and the water is flowing through these two pipes. In the first pipe, the pressure is $3.0 \times 10^4\ \text{N/m}^2$ and the speed of the water flowing is 5 m/s. If the diameters of the pipes are 4 cm and 6 cm, respectively, then what will be the speed and the pressure of the water in the second pipe? Density of the water is $10^3\ \text{kg/m}^3$.

Sol. According to the equation of continuity, we get

$$a_1 v_1 = a_2 v_2$$

$$\Rightarrow \quad \pi r_1^2 v_1 = \pi r_2^2 v_2$$

$$\therefore \quad v_2 = \left(\frac{r_1}{r_2}\right)^2 v_1$$

Given, $r_1 = \frac{4}{2} = 2\ \text{cm} = 2 \times 10^{-2}\ \text{m}$

$r_2 = \frac{6}{2} = 3\ \text{cm} = 3 \times 10^{-2}\ \text{m}$

$v_1 = 5\ \text{m/s}$

$$v_2 = \left(\frac{2}{3}\right)^2 \times 5 = 2.22\ \text{m/s}$$

Now, applying the Bernoulli's theorem

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$$p_2 = p_1 + \frac{1}{2}\rho(v_1^2 - v_2^2)$$

$$= 3.0 \times 10^4 + \frac{1}{2} \times 10^3 (5^2 - 2.22^2)$$

$$= 3 \times 10^4 + 500 \times 20.08$$

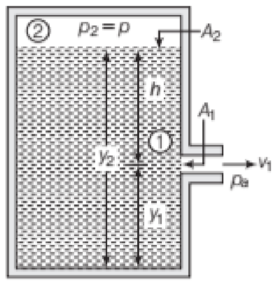
$$p_2 = 4 \times 10^4\ \text{N/m}^2$$

Applications of Bernoulli's Theorem

(i) **Speed of Efflux** (Torricelli's Law)

According to Torricelli's, velocity of efflux i.e. the velocity with which the liquid flows out of an orifice (i.e. a narrow hole) is equal to that which a freely falling body would acquire in falling through a vertical distance equal to the depth of orifice below the free surface of liquid.

Speed of efflux The word efflux means the outflow of a fluid as shown in figure. Consider a tank containing a liquid of density ρ with a small hole on its side at height y_1 from the bottom and y_2 be the height of the liquid surface from the bottom and p be the air pressure above the liquid surface.



Fluid flow from an orifice

If A_1, A_2 are the cross-sectional areas and v_1, v_2 are the velocities of liquid at point 1 and 2, respectively, then from the equation of continuity, we get

$$A_1 v_1 = A_2 v_2 \quad \text{or} \quad v_2 = \frac{A_1}{A_2} v_1$$

If $A_2 >> A_1$ so the liquid may be taken at rest at the top, i.e. $v_2 \approx 0$.

Applying the Bernoulli's theorem at points 1 and 2. The pressure $p_1 = p_a$ (the atmospheric pressure)

We get, $p_a + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p + \rho g y_2 \quad [\because v_2 \approx 0]$

or $$\frac{1}{2}\rho v_1^2 = \rho g (y_2 - y_1) + (p - p_a)$$

$$y_2 - y_1 = h$$

Hence, $$\frac{1}{2}\rho v_1^2 = \rho g h + (p - p_a)$$

Velocity of the liquid falls from orifice,

$$v_1 = \sqrt{2gh + \frac{2(p - p_a)}{\rho}}$$

(i) When the tank is open to the atmosphere,

$$p = p_a$$

and $$v_1 = \sqrt{2gh}$$

Thus, the **velocity of efflux** of a liquid is equal to the velocity which a body acquires in falling freely from the free liquid surface to the orifice. This result is called **Torricelli's law**.

Distance at which the stream strikes the floor

$$x = 2\sqrt{h y_1}$$

EXAMPLE |2| Water is Emerging Out from an Orifice

If the water emerges from an orifice in a tank in which the gauge pressure is $4 \times 10^5 \ N/m^2$ before the flow starts then, what will be the velocity of the water emerging out? Take density of water is $1000 \ kgm^{-3}$.

Sol. Here, $p = 4 \times 10^5 \ N/m^2$ and $\rho = 1000 \ kgm^{-3}$, $g = 10 \ m/s^2$

Apply $p = h\rho g$,

$$\Rightarrow \quad h = \frac{p}{\rho g} = \frac{4 \times 10^5}{1000 \times 10}$$

Velocity of efflux,

$$v = \sqrt{2gh} = \sqrt{\frac{2 \times 10 \times 4 \times 10^5}{1000 \times 10}}$$

$$= \sqrt{800} = 28.28 \ m/s$$

(ii) **Venturimeter**

It is a device used to measure the flow speed of incompressible fluid through a pipe. It is also called **flow meter** or **venturi tube**.

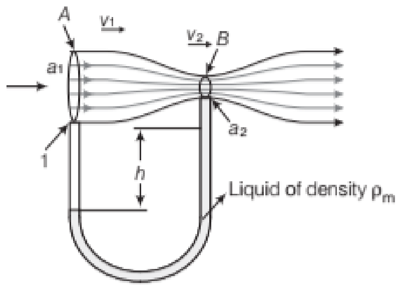
Construction It consists of a horizontal tube having wider opening of cross-section a_1 and a narrow neck of cross-section a_2. These two regions of the horizontal tube are connected to a manometer, containing a liquid of density ρ_m.

Working Let the liquid velocities be v_1 and v_2 at the wider and narrow region of the tube, respectively. Let p_1 and p_2 are liquid pressures at region A and B then,

According to the equation of continuity,

$$a_1 v_1 = a_2 v_2$$

or $$\frac{a_1}{a_2} = \frac{v_2}{v_1}$$



Venturimeter

Using Bernoulli's equation for horizontal flow of liquid with density ρ.

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

or $$p_1 - p_2 = \frac{1}{2}\rho (v_2^2 - v_1^2)$$

$$p_1 - p_2 = \frac{1}{2}\rho v_1^2 \left(\frac{v_2^2}{v_1^2} - 1\right)$$

$$p_1 - p_2 = \frac{1}{2}\rho v_1^2 \left(\frac{a_1^2}{a_2^2} - 1\right) \quad \ldots(i)$$

This pressure difference cause the liquid in the arm 2 of U tube connected at the narrow tube B to rise in comparison to other arm 1. The difference in height h of two arms of U tube measures the pressure difference.

$$p_1 - p_2 = h\rho_m g$$

$$h\rho_m g = \frac{1}{2}\rho v_1^2 \left(\frac{a_1^2}{a_2^2} - 1\right) \quad \text{[using Eq. (i)]}$$

From above equation

$$\text{Velocity of flow, } v_1 = \sqrt{\frac{2h\rho_m g}{\rho}}\left(\frac{a_1^2}{a_2^2}-1\right)^{-1/2}$$

It is speed of liquid in the wider tube.

The volume of the liquid flowing per second through the wider tube is

$$V = a_1 v_1 = a_1\sqrt{\frac{2h\rho_m g}{\rho}}\left(\frac{a_1^2}{a_2^2}-1\right)^{-1/2}$$

$$= a_1 a_2\sqrt{\frac{2h\rho_m g}{\rho(a_1^2-a_2^2)}} = a_1 a_2\sqrt{\frac{2(p_1-p_2)}{\rho(a_1^2-a_2^2)}}$$

So, $$\text{Volume of the liquid, } V = a_1 a_2\sqrt{\frac{2(p_1-p_2)}{\rho(a_1^2-a_2^2)}}$$

EXAMPLE |3| Blood Velocity

The flow of blood in a large artery of an anesthetised dog is diverted through a venturimeter. The wider part of the meter has a cross-sectional area equal to that of the artery, $a_1 = 8\text{mm}^2$. The narrower part has an area $a_2 = 4\text{mm}^2$. The pressure drop in the artery is 24 Pa. What is the speed of the blood in the artery? **[NCERT]**

Sol. The Bernoulli's equation for the horizontal flow is

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

By equation of continuity,

$$a_1 v_1 = a_2 v_2 \text{ or } v_2 = \frac{a_1 v_1}{a_2}$$

$$\therefore \quad p_1 - p_2 = \frac{1}{2}\frac{\rho a_1^2 v_1^2}{a_2^2} - \frac{1}{2}\rho v_1^2 = \frac{1}{2}\rho v_1^2\left[\left(\frac{a_1^2}{a_2^2}\right)-1\right]$$

Here, $p_1 - p_2 = 24\,\text{Pa}$

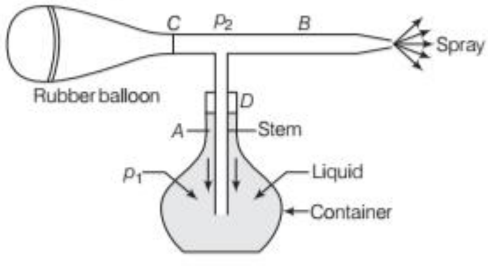
$\rho(\text{blood}) = 1.06\times10^3 \text{ kgm}^{-3}, a_1/a_2 = 8/4 = 2$

$$\therefore \quad v_1 = \sqrt{\frac{2(p_1-p_2)}{\rho\left(\frac{a_1^2}{a_2^2}-1\right)}}$$

$$= \sqrt{\frac{2\times24}{1.06\times10^3\times(2^2-1)}} = 0.1228 \text{ m/s}$$

(iii) Atomizer or Sprayer

It is based on the Bernoulli's principle. The essential parts of an atomizer are shown in figure. The forward stroke of the piston produces a stream of air past the end of the tube. The air flowing past the open end of the tube reduces the pressure on the liquid. So, the atmospheric pressure acting on the surface of liquid forces, the liquid into the tube D. As a result the liquid rises up in the vertical tube A. When it collides with the high speed air in tube B, it breaks up into fine spray.



Atomizer or sprayer

(iv) Blood Flow and Heart Attack

Consider a case, where a person suffering from heart attack problem, whose artery gets constricted due to the accumulation of plaque on its inner walls.

According to Bernoulli's principle, the pressure inside artery becomes low and the artery may collapse due to external pressure. The activity of heart is further increased in order to force the blood through that artery. As the blood rushes through that artery, the internal pressure once again drops due to same reason. This will be leading to a repeat collapse. This phenomenon is called **vascular flutter** which can be heard on a stethoscope. This may result in a heart attack.

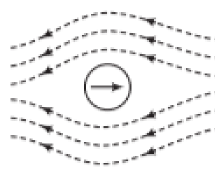
(v) Dynamic Lift

Dynamic lift is the force that acts on a body by virtue of its motion through a fluid. It is responsible for the curved path of a spinning ball and the lift of an aircraft wing.

(a) Ball Moving without Spin

When the velocity of the air above the ball is same as below the ball at the corresponding points resulting in zero pressure difference.

The air therefore, exerts no upward or downward force on the ball as shown in Fig. (a)

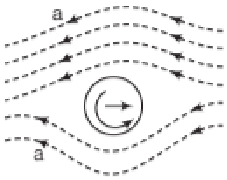


(a) Ball moving without spin

(b) Ball Moving with Spin

As the ball moves to the right, air rushes to the left with respect to the ball. Since the ball is spinning, it drags some air with it because of the roughness of its surface. The speed of air above the ball with respect to it is greater than below the ball. Hence, the pressure below the ball is greater than that above the ball. The force acts on the ball which makes it follow a curved path, as shown in Fig. (b).

The difference in lateral pressure, which causes a spinning ball to take a curved path which is curved towards the greater pressure side, is called **magnus effect**.



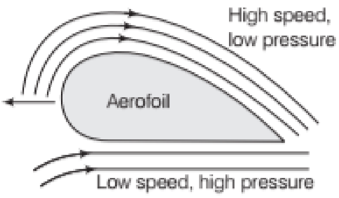
(b) Ball moving with spin

(c) Aerofoil, Lift of an Aircraft Wing

Aerofoil is a solid object shaped to provide an **upward dynamic lift** as it moves horizontally through air. This upward force makes aeroplane fly. The cross-section of the wing of an aeroplane looks like an aerofoil.

When the aeroplane moves through air, the air in the region above the wing moves faster than the air below as seen from the streamlines above the wing.

The difference in speed in the two regions makes the pressure in the region above lower than the pressure below the wing producing thereby a dynamic lift.



Aerofoil

EXAMPLE |4| A Boeing Aircraft

A fully loaded Boeing aircraft has a mass of 3.3×10^5 kg. Its total wing area is 500 m^2. It is in level flight with a speed of 960 km/h. (i) Estimate the pressure difference between the lower and upper surfaces of the wings (ii) Estimate the fractional increase in the speed of the air on the upper surface of the wing relative to the lower surface. [The density of air is $\rho = 1.2$ kgm^{-3}] **[NCERT]**

Sol. (i) The weight of the boeing aircraft is balanced by the upward force due to the pressure difference

$$\Delta p \times A = mg = 3.3\times10^5\times9.8$$

So, $\Delta p = \frac{mg}{A}$, $\Delta p = \frac{3.3\times10^5\times9.8}{500}$

$$= 6.46\times10^3\ N/m^2$$

$\therefore \quad \Delta p = 6.5\times10^3\ N/m^2$

(ii) Consider v_1 and v_2 are the speeds of air on lower and upper surfaces of the wings and p_1 and p_2 are the corresponding pressures.

From Bernoulli's principle

$$p_1+\frac{1}{2}\rho v_1^2 = p_2+\frac{1}{2}\rho v_2^2$$

$\therefore \quad p_1-p_2=\Delta p=\frac{\rho}{2}(v_2^2-v_1^2)$

$$(v_2-v_1)=\frac{2\Delta p}{\rho(v_2+v_1)}$$

Average speed, $v_{av}=\frac{v_2+v_1}{2}=960$ km/h $=267$ m/s

So, $\frac{v_2-v_1}{v_{av}}=\frac{\Delta p}{\rho v_{av}^2}=\frac{6.5\times10^3}{1.2\times(267)^2}=0.08=8\%$

Then, the speed above the wing needs to be only 8% higher than that below.

Roof can Blown off Without Damaging the House

During wind storm, the roofs of some huts are blown off without damaging the other parts of the house. The high wind blowing over the roof creates a low pressure p_2 in accordance with Bernoulli's principle.

The pressure p_1 below the roof is equal to the atmospheric pressure which is larger than p_2. This difference of pressure provides a vertical lift to the roof of hut. When lift is sufficient to overcome the gravity pull on the roof, the roof of the hut is blown off without causing any damage to the walls of hut.

VISCOSITY

When a solid body slides over another solid body, the force of friction opposes the relative motion of the solid bodies. In the same way, when a layer of fluid slides over another layer of the same fluid, a force of friction comes into play which is called **viscous force.** This force opposes the relative motion of the different layers of a fluid.

So, the tendency of fluids to oppose the relative motion of its layers is called **viscosity of fluid.** The backward dragging force called **viscous drag** or **viscous force.**

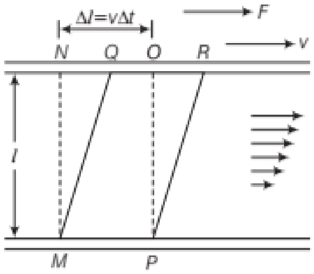
Cause of Viscosity

The velocities of the layers of the liquid increases uniformly from bottom to the top layer. For any layer of liquid, its lower layer pulls it backward while its upper layer pulls it forward direction. This type of flow is known as **laminar flow.**

Similar cases arise when the liquid, flowing in a pipe or a tube, then the velocity of the liquid is maximum along the axis of the tube and decreases gradually, as it moves towards the walls where it becomes zero.

Coefficient of Viscosity

Consider the flow of liquid as shown in figure. A portion of liquid which at some instant having the shape *MNOP* after the short interval of time (say Δt) the fluid is deformed and take the shape as *MQRP* since, the fluid has undergone the shear strain, stress in the solid is the force per unit area but in case of fluid, it depends on the rate of change of strain or strain rate.



The layer of fluid is deformed under some stress

Strain $\frac{\Delta l}{l}$ and the rate of change of strain is $\frac{\Delta l}{l\Delta t}=\frac{v}{l}$.

Hence, the coefficient of viscosity is defined as the ratio of shearing stress to the strain rate

$$\eta=\frac{F/A}{v/l}=\frac{Fl}{vA}$$

$$\boxed{\text{Coefficient of viscosity, } \eta=\frac{Fl}{vA}}$$

or it can be written as, $\eta=\dfrac{F/A}{\frac{dv}{dx}}$

Dimension of η

As, we know $\eta=\frac{F}{A}\cdot\frac{l}{v}$

$\therefore$ $$\eta=\frac{[MLT^{-2}]}{[L^2\,LT^{-1}]}[L]=[ML^{-1}T^{-1}]$$

Units of η

(i) In CGS system, the unit of η is **dyne-s/cm²** and it is called **poise**.

$$1\text{ poise}=\frac{1\text{ dyne}}{1\text{ cm}^2}\times\frac{1\text{cm}}{1\text{ cm/s}}$$

$$\mathbf{1\ poise = 1\ dyne\text{-}s/cm^2}$$

The coefficient of viscosity of a liquid is said to be 1 poise, if a tangential force of 1 dyne cm^{-2} of the surface is required to maintain a relative velocity of 1 cm s^{-1} between two layers of the liquid 1 cm apart.

(ii) The SI unit of η is **Ns/m²** or **kg/ms** and it is called **decapoise** or **poiseuille**.

$$1\text{ poiseuille}=\frac{1\text{N}}{1\text{ m}^2}\cdot\frac{1\text{ m}}{1\text{ m/s}}=1\text{ Ns/m}^2$$

The coefficient of viscosity of a liquid is said to be 1 poiseuille or decapoise if a tangential force of 1 Nm^{-2} of the surface is required to maintain a relative velocity of 1 ms^{-1} between two layers of the liquid 1 m apart.

(iii) Relation between poiseuille and poise

1 poiseuille or 1 decapoise = 10 poise

The coefficient of viscosity is a scalar quantity.

Relative Viscosity

$$\text{Relative viscosity of liquid}=\frac{\eta_{\text{liquid}}}{\eta_{\text{water}}}$$

Relative viscosity of bloods remains constant between 0° C and 37°C.

Note

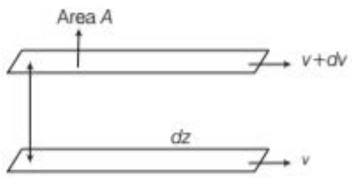
- Viscosity is just like friction force which converts kinetic energy into heat energy.
- No fluid has zero viscosity.
- Thin liquids like water, alcohol etc. are less viscous than thick liquids coal tar, blood, honey, glycerine etc.

Fluid friction force or force of viscosity

$$F=\eta A\frac{dv}{dz}$$

where, A = Area of layers

$\frac{dv}{dz}$ = Velocity gradient between two layers



EXAMPLE |5| Shearing Stress at a River Bed

Near the surface of the river, the velocity of water is 160 kmh^{-1}. Find the shearing stress between horizontal layers of water, if the river is 6 m deep and the coefficient of viscosity of water is 10^{-2} poise.

Sol. Given, $v=160\text{ km/h}=160\times\frac{5}{18}\text{ m/s}=44.44\text{ m/s}$

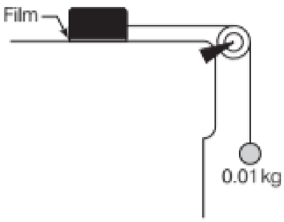
$l=6$ m and $\eta=10^{-2}$ poise $=10^{-3}$ Pa-s

$$\text{Shearing stress}=\frac{F}{A}=\eta\frac{v}{l}=10^{-3}\times\frac{44.44}{6}$$

Shearing stress $=7.407\times10^{-3}\ Nm^{-2}$

EXAMPLE |6| Effect of Viscous Force

A metal block of area 0.10 m^2 is connected to a 0.01 kg mass via a string that passes over an ideal pulley (considered massless and frictionless), as in figure. A liquid with a film thickness of 0.30 mm is placed between the block and the table. When released the block moves to the right with a constant speed of 0.085 ms^{-1}. Find the coefficient of viscosity of the liquid.



[NCERT]

Sol. Given, $m = 0.010$ kg, $g = 9.8$ m/s^2, $A = 0.10$ m^2

$$F = T = mg = 0.010 \times 9.8 = 9.8 \times 10^{-2} \text{ N}$$

$$\text{Shear stress on the fluid} = \frac{F}{A} = \frac{9.8 \times 10^{-2}}{0.10} = 0.98 \text{ N/m}^2$$

$$\text{The strain rate,} \quad \eta = \frac{\text{Stress}}{\text{Strain rate}}$$

$$\text{Strain rate} = \frac{v}{l} = \frac{0.085}{0.30 \times 10^{-3}}$$

Now, the coefficient of viscosity of the liquid.

$$\eta = \frac{F\,l}{Av} = \frac{9.8 \times 10^{-2} \times 0.30 \times 10^{-3}}{0.10 \times 0.085}$$

$$\eta = 3.45 \times 10^{-3} \text{ Pa-s}$$

Difference between Viscous Force and Solid Friction

Viscous Force	Solid Friction
Viscous force is directly proportional to the area of layers in contact.	Solid friction is independent of the area of the surfaces in contact.
It is directly proportional to the relative velocity between the two liquid layers.	It is independent of the relative velocity between two solid surfaces.
It is independent of the normal reaction between the two liquid layers.	It is directly proportional to the normal reaction between the surfaces in contact.

Effect of Temperature on the Viscosity

The viscosity of liquids decreases with increase in temperature and increases with decrease in temperature.

i.e. $$\eta \propto \frac{1}{\sqrt{T}}$$

On the other hand, the viscosity of gases increases with the increase in temperature and *vice-versa*.

i.e. $$\eta \propto \sqrt{T}$$

CRITICAL VELOCITY

The critical velocity of a liquid is that limiting value of its velocity of flow upto which the flow is streamlined and above which the flow becomes turbulent.

The critical velocity v_c of a liquid flowing through a tube depends on

(i) coefficient of viscosity of the liquid (η)

(ii) density of the liquid (ρ)

(iii) radius of the tube (r)

Consider, $\quad v_c = k\eta^a \rho^b r^c$

where, k is a dimensionless constant. Writing the above equation in dimensional form, we get

$$[M^0LT^{-1}] = [ML^{-1}T^{-1}]^a [ML^{-3}]^b [L]^c$$

$$[M^0LT^{-1}] = [M^{a+b}L^{-a-3b+c}T^{-a}]$$

Compare the powers of M, L and Ton both sides, we get

$$a + b = 0$$

$$-a - 3b + c = 1$$

$$-a = -1$$

On solving, we get,

$$a = 1, b = -1, c = -1$$

$$\therefore \quad v_c = k\eta\rho^{-1}r^{-1} = \frac{k\eta}{\rho r}$$

We get,

$$\boxed{\text{Critical velocity, } v_c = \frac{k\eta}{\rho r}}$$

For the flow to the streamline, value of v_c should be as large as possible. For this, η should be large, ρ and r should be small. So, we conclude that

(i) The flow of liquids of higher viscosity and lower density through narrow pipes tends to be **streamlined**.

(ii) The flow of liquids of lower viscosity and higher density through broad pipes tends to become **turbulent** because in that case the critical velocity will be very small.

STOKE'S LAW

When a small spherical body falls through a viscous fluid at rest, the layers of fluid in contact with the body are dragged alongwith it. But the layers of the fluid away from the body are at rest. This produces a relative motion between different layers of the fluid.

As a result, a **backward dragging** force (i.e. viscous force) comes into play, which opposes the motion of the body.

This backward dragging force increases with the increase in velocity of the moving body. Falling of a raindrop, swinging of a pendulum bob are the examples of such type of motion.

Sir George G Stokes (1819–1903), an English scientist found that the backward dragging force F acting on a small spherical body of radius r, moving through a fluid of coefficient of viscosity η, with velocity v is given by

$$\boxed{\text{Dragging force, } F = 6\pi\eta rv}$$

This is called **Stoke's law** of viscosity.

He observed that viscous drag (F) depends upon

(i) coefficient of viscosity (η) of the fluid.

(ii) velocity (v) of the body.

(iii) radius (r) of the spherical body.

Let $\qquad F = k\eta^a v^b r^c \qquad$ ...(i)

As $\qquad [F] = [MLT^{-2}], [\eta] = [ML^{-1}T^{-1}]$

$\qquad [v] = [LT^{-1}], [r] = [L]$

So, $\qquad [MLT^{-2}] = [ML^{-1}T^{-1}]^a [LT^{-1}]^b [L]^c$

$\qquad [MLT^{-2}] = [M^a L^{-a+b+c} T^{-a-b}]$

Compare powers both sides of M, L, T, we get

$$a = 1 \qquad \text{...(ii)}$$

$$-a + b + c = 1 \qquad \text{...(iii)}$$

$$-a - b = -2 \qquad \text{...(iv)}$$

or $\qquad a + b = 2 \qquad$ ...(v)

From Eqs. (iii) and (v), we get

$$c = 1, \quad b = 1$$

Substituting these values in Eq. (i), we get

$$F = 6\pi\eta rv$$

where, the value of k was found to be 6π experimentally.

Note

- This law is used in the determination of electronic charge with the help of Millikan's experiment.
- This law accounts the formation of clouds.
- This law accounts, why the speed of rain drops is less than that of a body falling freely with a constant velocity from the height of clouds.
- This law helps a man coming down with the help of a parachute.

EXAMPLE |7| Dragging Force on Rain Droplets

Consider a drop of rain having radius 0.4 mm and terminal velocity 2 m/s. Find the viscous force on the rain drops, if viscosity of air is 18×10^{-5} dyne cm^{-2}s.

The Stoke's law is an interesting example of retarding force which is proportional to velocity.

Sol. Given, radius, $r = 0.4 \text{ mm} = 0.4 \times 10^{-3}$ m

Terminal velocity, $v = 2 \text{ ms}^{-1}$

and viscosity of air, $\eta = 18 \times 10^{-5}$ dyne cm^{-2}s

$= 18 \times 10^{-6}$ Pa-s

According to Stoke's law, the viscous force $F = 6\pi\eta rv$

$= 6 \times 3.142 \times 18 \times 10^{-6} \times 0.4 \times 10^{-3} \times 2$

$F = 2.71 \times 10^{-7}$ N

TERMINAL VELOCITY

The maximum constant velocity acquired by a body while falling through a viscous fluid is called its **terminal velocity**.

Consider an example of raindrop in air. It accelerates initially due to gravity. The force of viscosity increases as the velocity of the body increases. A stage is reached, when the true weight of the body becomes just equal to the sum of the upthrust and the viscous force. Then, the body begins to fall with a constant velocity called **terminal velocity**.

Suppose a sphere of density ρ falls through a liquid of density σ. When the body attains terminal velocity v, upward thrust + force of viscosity = weight of the spherical body

Upward thrust = The weight of the fluid displaced

$$= \frac{4}{3}\pi r^3 \sigma g$$

Force of viscosity $= 6\pi\eta rv$, and weight of the spherical body

$$= \frac{4}{3}\pi r^3 \rho g \qquad [\because mV\rho]$$

Thus, $\quad \frac{4}{3}\pi r^3 \sigma g + 6\pi\eta rv = \frac{4}{3}\pi r^3 \rho g$

or $\qquad 6\pi\eta rv = \frac{4}{3}\pi r^3 (\rho - \sigma) g$

or $\qquad \boxed{\text{Terminal velocity, } v = \frac{2}{9} \cdot \frac{r^2(\rho - \sigma) g}{\eta}}$

where, $\quad r$ = radius of the spherical body,

$\qquad v$ = terminal velocity

and $\quad \eta$ = coefficient of viscosity of fluid

EXAMPLE |8| Velocity of a Spherical Droplet

If 27 drops of rain were to be combine to form one new large spherical drop, then what should be the velocity of this large spherical drop? Consider the terminal velocity of 27 drops of equal size falling through the air is 0.20 ms^{-1}.

Sol. Let, the radius of the small drop is r and that of big drop is R. The volume of the big drop = 27 × volume of each small drop

$$\frac{4}{3}\pi R^3 = 27 \times \frac{4}{3}\pi r^3 \Rightarrow R = 3r$$

Let, the terminal velocities of small and big drop are v_1 and v_2, respectively. Then,

$$v = \frac{2r^2(\rho - \sigma)g}{9\eta} \Rightarrow v \propto r^2$$

Hence, $\frac{v_2}{v_1} = \frac{R^2}{r^2} \Rightarrow v_2 = v_1 \times \frac{R^2}{r^2} = 0.2\left(\frac{3r}{r}\right)^2 = 0.2 \times 9$

$v_2 = 1.8$ m/s

REYNOLD'S NUMBER

Osborne Reynolds (1842-1912) observed that turbulent flow is less likely for viscous fluid flowing at low rates. He defined a dimensionless parameter whose value decides the nature of flow of a liquid through a pipe, i.e. whether a flow will be steady or turbulent, it is given by

$$\text{Reynold's number, } R_e = \frac{\rho v D}{\eta}$$

where, ρ = density of the liquid,
v = velocity of the liquid,
η = coefficient of viscosity of the liquid
and D = diameter of the pipe

(i) If R_e lies between 0 and 2000, then liquid flow is streamline or laminar.
(ii) If $R_e > 3000$, then liquid flow is turbulent.
(iii) If R_e lies between 2000 and 3000, then flow of liquid is unstable, it may change from laminar to turbulent and *vice-versa*.

The exact value at which turbulent sets in a fluid is called **critical Reynold's number**.

In another form, R_e can also be written as

$$R_e = \frac{\rho v D}{\eta} = \frac{\rho v^2}{\left(\eta \frac{v}{D}\right)} = \frac{\rho A v^2}{\eta \frac{Av}{D}} = \frac{\text{Inertial force}}{\text{Force of viscosity}}$$

Hence, Reynold's number represents the ratio of the inertial force per unit area to the viscous force per unit area.

EXAMPLE |9| Finding the Nature of the Flow

The flow rate of water is 0.58 L/mm from a tap of diameter of 1.30 cm. After some time, the flow rate is increased to 4 L/min. Determine the nature of the flow for both the flow rates. The coefficient of viscosity of water is 10^{-3} Pa - s and the density of water is 10^3 kg/m^3.

Sol. Given, diameter, $D = 1.30 \text{ cm} = 1.3 \times 10^{-2}$ m

Coefficient of viscosity of water, $\eta = 10^{-3}$ Pa-s

Density of water, $\rho = 10^3$ kg/m^3

The volume of the water flowing out per second is

$$V = vA = v \times \pi r^2 = v\pi\frac{D^2}{4}$$

$\therefore$ Speed of flow, $v = \frac{4V}{\pi D^2}$

Reynold's number, $R_e = \frac{\rho v D}{\eta} = \frac{4\rho V}{\eta \pi D}$

Case I When $V = 0.58$ L/min

$$= \frac{0.58 \times 10^{-3} \text{m}^3}{1 \times 60 \text{ s}}$$
$$= 9.67 \times 10^{-6} \text{m}^3\text{s}^{-1}$$

$$R_e = \frac{4 \times 10^3 \times 9.67 \times 10^{-6}}{10^{-3} \times 3.14 \times 1.3 \times 10^{-2}} = 948$$

$\because R_e < 1000$, so the flow is steady or streamline

Case II When $V = 4$ L/min

$$= \frac{4 \times 10^{-3}}{60} \text{m}^3\text{s}^{-1} = 6.67 \times 10^{-5} \text{m}^3\text{s}^{-1}$$

$$R_e = \frac{4 \times 10^3 \times 6.67 \times 10^{-5}}{10^{-3} \times 3.14 \times 1.3 \times 10^{-2}} = 6536$$

$\because R_e > 3000$, so the flow will be turbulent.

Poiseuille's Formula

The volume of a liquid flowing out per second through a horizontal capillary tube of length l, radius r, under a pressure difference p applied across its ends is given by

$$V = \frac{Q}{t} = \frac{\pi p r^4}{8\eta l}$$

This formula is called **Poiseuille's** formula.

TOPIC PRACTICE 3

OBJECTIVE Type Questions

1. In a streamline flow,
(a) the speed of a particle always remains same
(b) the velocity of a particle always remains same
(c) the kinetic energies of all the particles arriving at a given point are the same
(d) the potential energies of all the particles arriving at a given point are the same

Sol. (b) Both velocity and direction of flow remain same.

2. Two water pipes of diameters 2 cm and 4 cm are connected with the main supply line. The velocity of flow of water in the pipe of 2 cm diameter is
(a) 4 times that in the other pipe
(b) $\frac{1}{4}$ times that in the other pipe
(c) 2 times that in the other pipe
(d) $\frac{1}{2}$ times that in the other pipe

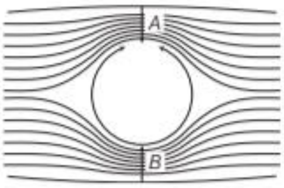
Sol. (a) From equation of continuity, av = constant

$$d_A = 2\text{ cm} \quad \text{and} \quad d_B = 4\text{ cm}$$

$$\therefore \quad r_A = 1\text{ cm} \quad \text{and} \quad r_B = 2\text{ cm}$$

$$\therefore \quad \frac{v_A}{v_B} = \frac{a_B}{a_A} = \frac{\pi(r_B)^2}{\pi(r_A)^2} = \left(\frac{2}{1}\right)^2 \Rightarrow v_A = 4v_B$$

3. A ball is moving without spinning in a straight line through a fluid (as shown)



If p_A and p_B are pressure values at A and B, then

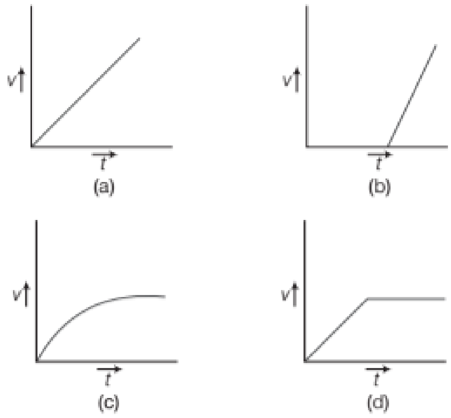
(a) $p_A < p_B$ (b) $p_B < p_A$

(c) $p_A \times p_B = 1$ (d) $p_A / p_B = 1$

Sol. (d) As the ball is not spinning, By Bernoulli's theorem,

$$p_A = p_B \Rightarrow \frac{p_A}{p_B} = 1$$

4. A tall cylinder is filled with viscous oil. A round pebble is dropped from the top with zero initial velocity. From the plot shown in figure, indicate the one that represents the velocity (v) of the pebble as a function of time (t). **[NCERT Exemplar]**



Sol. (c) When the pebble is falling through the viscous oil the viscous force is

$$F = 6\pi\eta r v$$

As the force is variable, hence acceleration is also variable so v-t graph will not be straight line.First velocity increases and then becomes constant known as terminal velocity.

5. The coefficient of viscosity for hot air is

(a) greater than the coefficient of viscosity for cold air

(b) smaller than the coefficient of viscosity for cold air

(c) same as the coefficient of viscosity for cold air

(d) increases or decreases depending on the external pressure

Sol. (a) For gases, viscosity increases with temperature.

VERY SHORT ANSWER Type Questions

6. Can two streamlines cross each other, why?

Sol. Two streamlines can never cross each other because if they cross them at the point of intersection, there will be two possible direction of flow of fluid which is impossible for streamlines.

7. Why does the velocity increase when liquid flowing in a wider tube enters a narrow tube? **[NCERT]**

Sol. This is due to equation of continuity, $a_1 v_1 = a_2 v_2$

$$\because \quad a_1 > a_2$$

$$\therefore \quad v_2 > v_1$$

8. Can Bernoulli's equation be used to describe the flow of water through a rapid in a river? Explain.

Sol. No, Bernoulli's equation cannot be used to describe the flow of water through a rapid in a river because Bernoulli's equation can be utilised only for streamline flow.

9. Does it matter if one uses gauge instead of absolute pressures in applying Bernoulli's equation? Explain.

Sol. No, it does not matter if one uses gauge instead of absolute pressures in applying Bernoulli's equation unless the atmospheric pressure at the two points where, Bernoulli's equation is applied are significantly different.

10. The height of water level in a tank is $H = 96$ cm. Find the range of water stream coming out of a hole at depth $H/4$ from upper surface of water.

Sol. The depth of hole below the upper surface of water is

$$h = \frac{H}{4} = \frac{96}{4} = 24\text{ cm}$$

The height of hole from ground is,

$$h' = 96 - 24 = 72\text{ cm}$$

$$\text{Horizontal range} = 2\sqrt{hh'} = 2\sqrt{24 \times 72} = 48\sqrt{3}\text{ cm}$$

11. A hot liquid moves faster than a cold liquid. Why?

Sol. The viscosity of liquid decreases with the increase in temperature. Therefore, viscosity of hot liquid is less than that of cold liquid. Due to this, hot liquid moves faster than the cold liquid.

12. On what factors does the critical speed of fluid flow depend?

Sol. The critical speed of a fluid depends on (a) diameter of tube, (b) density of fluid, (c) coefficient of viscosity of the fluid.

SHORT ANSWER Type Questions

13. Fill in the blanks using the word(s) from the list appended with each statement.

(i) Surface tension of liquids generally......with temperature. (increases/decreases)

(ii) Viscosity of gases......with temperature, whereas, viscosity of liquids......with temperature. (increases/decreases)

(iii) For solids with elastic modulus of rigidity, the shearing force is proportional to......while for fluids, it is proportional to......(shear strain/rate of shear strain)

(iv) For a fluid in a steady flow, the increase in flow speed at a constriction follows from......while the decrease of pressure there follows from......(conservation of mass/Bernoulli's principle).

(v) For the model of a plane in a wind tunnel, turbulence occurs at a......speed that the critical speed for turbulence for an actual plane (greater/smaller) **[NCERT]**

Sol. (i) decreases
(ii) increases, decreases
(iii) shear strain, rate of shear strain
(iv) conservation of mass, Bernoulli's principle
(v) greater

14. In deriving Bernoulli's equation, we equated the work done on the fluid in the tube to its change in the potential and kinetic energy.

(i) How does the pressure change as the fluid moves along the tube, if dissipative forces are present?

(ii) Do the dissipative forces become more important as the fluid velocity increases? Discuss qualitatively. **[NCERT]**

Sol. (i) If dissipative forces are present, then a part of pressure energy is utilised in overcoming these forces. Due to which, the pressure decreases as the fluid moves along the tube.

(ii) Yes, the dissipative forces become more important as the fluid velocity increases.

The viscous drag is given by

$$F = -\eta A \frac{dv}{dx}$$

As the velocity of fluid increases, the velocity gradient increases and hence, viscous drag increases i.e. dissipative force also increases.

15. Figs. (a) and (b) refer to the steady flow of a non-viscous liquid. Which of two figures is incorrect Why? **[NCERT]**

(a) (b)

Sol. Fig. (a) is incorrect. According to equation of continuity, the speed of liquid is larger at smaller area.

From Bernoulli's theorem due to larger speed, the pressure will be lower at smaller area and therefore, height of liquid column will also be at lesser height, while in Fig. (a) height of liquid column at narrow area is higher.

16. At what speed will the velocity head of stream of water be 40 cm?

Sol. Given, $h = 40$ cm,

$$g = 980\ \text{cm/s}^2$$

We know that velocity head, $h = \frac{v^2}{2g}$

$$\therefore \quad v = \sqrt{2gh} = \sqrt{2 \times 980 \times 40}$$

$$= 280\ \text{cms}^{-1}$$

17. In Millikan's oil drop experiment, what is the terminal speed of an uncharged drop of radius 2.0×10^{-5} m and density 1.2×10^3 kg/m^3? Take the viscosity of air at the temperature of the experiment to be 1.8×10^{-5} Pa-s. How much is the viscous force on the drop at that speed? Neglect buoyancy of the drop due to air. **[NCERT]**

Sol. Given, radius of drop $(r) = 2.0 \times 10^{-5}$ m

Density of oil $(\rho) = 1.2 \times 10^3$ kg/m^3

Viscosity of air $(\eta) = 1.8 \times 10^{-5}$ Pa-s

$$\text{Terminal velocity, } v = \frac{2}{9} \frac{r^2(\rho - \rho_0)g}{\eta}$$

$$= \frac{2}{9} \times \frac{(2 \times 10^{-5})^2 \times (1.2 \times 10^3 - 0) \times 9.8}{1.8 \times 10^{-5}}$$

$$= 5.8 \times 10^{-2}\ \text{m/s}$$

$\therefore$ Viscous force on the drop, (According to Stoke's law)

$$F = 6\pi\eta r v = 6 \times 3.14 \times 1.8 \times 10^{-5} \times 2 \times 10^{-5} \times 5.8 \times 10^{-2}$$

$$= 3.93 \times 10^{-10}\ \text{N}$$

18. The terminal velocity of a copper ball of radius 2.0 mm falling through a tank of oil at 20°C is 6.5 cms^{-1}. Compute the viscosity of the oil at 20°C. Density of oil is 1.5×10^3 kg m^{-3}, density of copper is 8.9×10^3 kg m^{-3}. **[NCERT]**

Sol. Given, $v_t = 6.5 \times 10^{-2}$ m/s

$$r = 2 \times 10^{-3} \text{ m}, g = 9.8 \text{ m/s}^2$$

$$\rho = 8.9 \times 10^3 \text{ kg/m}^3, \sigma = 1.5 \times 10^3$$

$$\rho - \sigma = 8.9 \times 10^3 - 1.5 \times 10^3$$

$$= 7.4 \times 10^3 \text{ kg/m}^3$$

As, we know terminal velocity

i.e. $$v_t = \frac{2}{9}\frac{r^2(\rho - \sigma)}{\eta} g$$

$$\eta = \frac{2}{9} \times \frac{(2 \times 10^{-3})^2 \times 9.8}{(6.5 \times 10^{-2})} \times 7.4 \times 10^3$$

$$\eta = 9.9 \times 10^{-1} \text{ kg/ms}$$

19. The velocity of water in a river is 18 kmh^{-1} near the surface. If the river is 5 m deep, find the shearing stress between horizontal layers of water. The coefficient of viscosity of water 10^{-2} poise.

Sol. As the velocity of water at the bottom of the river is zero,

$$dv = 18 \text{ kmh}^{-1} = 18 \times \frac{5}{18} = 5 \text{ ms}^{-1}$$

Also, $dx = 5$ m, $\eta = 10^{-2}$ poise $= 10^{-3}$ Pa-s

Force of viscosity, $F = \eta A \frac{dv}{dx}$

We know that, shearing stress $= \frac{F}{A}$

$$\Rightarrow \quad \frac{F}{A} = \eta \frac{dv}{dx} = \frac{10^{-3} \times 5}{5} = 10^{-3} \text{ Nm}^{-2}$$

20. What should be the average velocity of water in a tube of radius 0.005 m so that the flow is just turbulent? The viscosity of water is 0.001 Pa-s.

Sol. Here, $r = 0.005$ m, diameter $D = 2r = 0.010$ m

$$\eta = 0.001 \text{ Pa-s}, \rho = 1000 \text{ kgm}^{-3}$$

For flow to be just turbulent, $R_e = 3000$

$$\therefore \quad v = \frac{R_e \eta}{\rho D} = \frac{3000 \times 0.001}{1000 \times 0.010} = 0.3 \text{ ms}^{-1}$$

LONG ANSWER Type I Questions

21. In a test experiment on a model aeroplane in a wind tunnel, the flow of speeds on the upper and lower surfaces of the wings are 70 m/s and 63 m/s respectively. What is the lift on the wings, if its area is 2.5 m^2? Take the density of air to be 1.3 kg / m^3. **[NCERT]**

Sol. Let the lower and upper surface of the wings of the aeroplane be at the same height h and speeds of air on the upper and lower surfaces of the wings be v_1 and v_2.

Speed of air on the upper surface of the wings

$$v_1 = 70 \text{ m/s}$$

Speed of air on the lower surface of the wings

$$v_2 = 63 \text{ m/s}$$

Density of the air, $\rho = 1.3 \text{ kg/m}^3$

Area, $A = 2.5 \text{ m}^2$

According to Bernoulli's theorem,

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h$$

or $$p_2 - p_1 = \frac{1}{2}\rho(v_1^2 - v_2^2)$$

$\therefore$ Lifting force acting on the wings,

$$F = (p_2 - p_1) \times A$$

$$= \frac{1}{2}\rho(v_1^2 - v_2^2) \times A \quad [\because \text{force} = \text{pressure} \times \text{area}]$$

$$= \frac{1}{2} \times 1.3 \times [(70)^2 - (63)^2] \times 2.5$$

$$= \frac{1}{2} \times 1.3 [4900 - 3969] \times 2.5$$

$$= \frac{1}{2} \times 1.3 \times 931 \times 2.5 = 1.51 \times 10^3 \text{ N}$$

22. Air is streaming past a horizontal air plane wing such that its speed is 120 ms^{-1} over the upper surface and 90 ms^{-1} at the lower surface. If the density of air is 1.3 kgm^{-3}, find the difference in pressure between the top and bottom of the wing. If wing is 10 m long and has an average width of 2 m, calculate the gross lift of the wing.

Sol. Given, $v_2 = 120$ m/s, $v_1 = 90$ m/s, $\rho_a = 1.3 \text{ kg/m}^3$,

$h_1 = 10$ m, $a_1 = 10 \times 2 = 20 \text{ m}^2$

According to Bernoulli's theorem,

$$\frac{p_1}{\rho} + gh_1 + \frac{1}{2}v_1^2 = \frac{p_2}{\rho} + gh_2 + \frac{1}{2}v_2^2$$

For the horizontal flow, $h_1 = h_2$

$$\therefore \quad \frac{p_1}{\rho} + \frac{1}{2}v_1^2 = \frac{p_2}{\rho} + \frac{1}{2}v_2^2$$

Given, $v_1 = 90$ m/s, $v_2 = 120$ m/s , $\rho = 1.3 \text{ kg/m}^3$

$$\therefore \quad \frac{p_1 - p_2}{\rho} = \frac{1}{2}(v_2^2 - v_1^2)$$

$$(p_1 - p_2) = \frac{\rho(v_2^2 - v_1^2)}{2}$$

$$= 1.3 \times \frac{(14400 - 8100)}{2} = \frac{1.3 \times 6300}{2}$$

$$p_1 - p_2 = 4.095 \times 10^3 \text{ N/m}^2$$

It is the pressure difference between the top and the bottom of the wing.

Gross lift of wing $= (p_1 - p_2) \times$ Area of the wing

$$= 4.095 \times 10^3 \times 10 \times 2$$

$$= 8.190 \times 10^4 \text{ N}$$

23. A venturimeter is connected to two points in the mains where its radii are 20cm and 10cm, respectively, and the levels of water column in the tubes differ by 10 cm. How much water flows through the pipe per minute?

Sol. As we know that,

The volume of water flowing per second

$$V = a_1 a_2 \sqrt{\frac{2h\rho_m g}{\rho(a_1^2 - a_2^2)}}$$

$$\because \quad V = a_1 a_2 \sqrt{\frac{2hg}{a_1^2 - a_2^2}}$$

$$\because \quad r_1 = 20\text{cm}, \; a_1 = \pi r_1^2 = \pi(20)^2\,\text{cm}^2$$

$$r_2 = 10\,\text{cm}, \; a_2 = \pi r_2^2 = \pi(10)^2\,\text{cm}^2$$

$$r_1 = 10\text{cm}, \; g = 980\text{cm}/s^2$$

$$\therefore \quad V = \pi^2(20)^2 \cdot (10)^2 \sqrt{\frac{2 \times 10 \times 980}{\pi^2((20)^4 - (10)^4}}\ \text{c.c/sec}$$

$$= \frac{175.93 \times 10^3}{\sqrt{15}}\text{c.c/sec}$$

$$= \frac{175.93 \times 10^3}{\sqrt{15}} \times 60\text{c.c/min}$$

$= 2728.7$ litres/min

24. (i) What is the largest average velocity of blood flow in an artery of radius 2×10^{-3}m, if the flow must remain laminar?

(ii) What is the corresponding flow rate? (Take, viscosity of blood to be 2.084×10^{-3}Pa-s) **[NCERT]**

Sol. Given, radius of artery $(r) = 2 \times 10^{-3}$ m

$\therefore$ Diameter of artery $D = 2r = 4 \times 10^{-3}$ m

Density of whole blood $(\rho) = 1.06 \times 10^3\,\text{kg/m}^3$

Coefficient of viscosity of blood $(\eta) = 2.084 \times 10^3$ Pa-s

For laminar flow, maximum value of Reynold's number

$$R_e = 2000$$

(i) Critical velocity $(v_c) = \dfrac{R_e \eta}{\rho D}$

$$= \frac{2000 \times 2.084 \times 10^3}{1.06 \times 10^3 \times 4 \times 10^{-3}}$$

$$= 9.83 \times 10^5\,\text{m/s}$$

(ii) Flow rate of blood = Volume of blood flowing per second

$$= Av_c$$

$$= \pi r^2 \times v_c$$

$$= 3.14 \times (2 \times 10^{-3})^2 \times 9.83 \times 10^5$$

$$= 12.35\ \text{m}^3/\text{s}$$

25. Show that if n equal rain droplets falling through air with equal steady velocity of $10\ \text{cms}^{-1}$ coalesce, the resultant drop attains a new terminal velocity of $10\,n^{2/3}\text{cms}^{-1}$.

Sol. Volume of a bigger drop

$= n \times$ Volume of a smaller droplet

or $\quad \frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3$ or $R^3 = nr^3$ or $R = n^{1/3} r$

Terminal velocity of a small droplet is given by

$$v_s = \frac{2}{9}\frac{r^2}{\eta}(\rho - \rho')\,g \qquad \text{...(i)}$$

Terminal velocity of a bigger drop is given by

$$v_b = \frac{2}{9}\frac{R^2}{\eta}(\rho - \rho')\,g \qquad \text{...(ii)}$$

Dividing Eq. (ii) by Eq. (i), we get $\dfrac{v_b}{v_s} = \dfrac{R^2}{r^2}$

But $\quad R = n^{1/3} r$ and $v_s = 10$ cm/s

$$v_b = v_s \times \left(\frac{R^2}{r^2}\right) = 10 \times \frac{n^{2/3} r^2}{r^2}$$

$$v_b = 10\,n^{2/3}\,\text{cm/s}$$

26. The flow rate of water from a tap of diameter 1.25 cm is 0.48 L/min. The coefficient of viscosity of water is 10^{-3}Pa-s. After some time the flow rate is increased to 3 L/min. Characterise the flow for both the flow rates. **[NCERT]**

Sol. Let the speed of the flow be v.

Given, diameter of tap $= d = 1.25$ cm

Volume of water flowing out per second.

$$Q = v \times \frac{\pi d^2}{4}$$

$$\Rightarrow \quad v = \frac{4Q}{d^2 \pi}$$

Estimate Reynold's number,

$$R_e = \frac{4\rho Q}{\pi d \eta}$$

$$Q = 0.48\ \text{L/min} = 8 \times 10^{-3}\,\text{L/s} = 8 \times 10^{-6}\,\text{m}^3/\text{s}$$

$$R_e = \frac{4 \times 10^3 \times 8 \times 10^{-6}}{3.14 \times 1.25 \times 10^{-2} \times 10^{-3}}$$

$R_e = 815$ [i.e. below 1000, the flow is steady]

After some time, when

$$Q = 3\ \text{L/min} = 5 \times 10^{-5}\,\text{m}^3/\text{s},$$

$$R_e = \frac{4 \times 10^3 \times 5 \times 10^{-5}}{3.14 \times 1.25 \times 10^{-2} \times 10^{-3}} = 5095$$

$\therefore$ The flow will be turbulent.

27. The cylindrical tube of a spray pump has a cross-section of 8.0 cm^2 one end of which has 40 fine holes each of diameter 1.00 mm. If the liquid flow inside the tube is 1.5 m/min, what is the speed of ejection of the liquid through the holes? [NCERT]

Sol. Area of cross-section of tube $(A) = 8\ cm^2 = 8 \times 10^{-4} m^2$

Number of holes, $N = 40$

Diameter of each hole, $2r = 1.0$ mm

$\therefore$ Radius of each hole, $r = 0.5\ mm = 5 \times 10^{-4}$ m

Velocity of liquid flow in tube $= 1.5\ m/min = \frac{1.5}{60}\ m/s$
$= 2.5 \times 10^{-2}\ m/s$

Total area of holes $= N \times \pi r^2 = 40 \times 3.14 \times (5 \times 10^{-4})^2$
$= 3.14 \times 10^{-5} m^2$

From equation of continuity,

$$A_1 v_1 = A_2 v_2$$

or
$$v_2 = \frac{A_1 v_1}{A_2} = \frac{8 \times 10^{-4} \times 2.5 \times 10^{-2}}{3.14 \times 10^{-5}}$$
$$= \frac{20}{3.14} \times 10^{-1} = 0.64\ m/s$$

28. A cylindrical vessel filled with water upto a height of 2 m stands on a horizontal plane. The side wall of the vessel has a plugged circular hole touching the bottom. Find the minimum diameter of the hole so that the vessel begin to move on the floor, if the plug is removed. The coefficient of friction between the bottom of the vessel and the plane is 0.4 and total mass of water plus vessel is 100 kg.

Sol. Velocity of efflux through the hole, $v = \sqrt{2gh}$

$\because$ Distance moved by water in one second $v = \sqrt{2gh}$

$\therefore$ Rate of the momentum $= (\rho A \sqrt{2gh})(\sqrt{2gh}) = 2\ ghA\rho$

According to Newton's second law of motion,

Force due to the velocity of efflux $= 2\ gh\ A\rho$.

Now, according to Newton's third law of motion,

Force on the vessel = Rate of the momentum

Force on the vessel $= 2\ gh\ A\rho$

The vessel will move, if force on the vessel = force of friction

or $2gh\ A\rho = \mu Mg$

or
$$A = \frac{\mu M}{2h\rho} = \frac{0.4 \times 100}{2 \times 2 \times 1000} = \frac{1}{100}$$

Since, the hole is circular.

$$A = \pi r^2 = \frac{\pi D^2}{4}$$

$\therefore$
$$D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \times 1}{100 \times 3.14}} = 0.113\ m$$

So, diameter of a hole $D = 0.113$ m

LONG ANSWER Type II Questions

29. Explain why?

(i) To keep a piece of paper horizontal, you should blow over, not under it.

(ii) When we try to close a water tap with our fingers, fast jets of water gush through the opening between our fingers.

(iii) The size of the needle of a syringe controls flow rate better than the thumb pressure exerted by a doctor while administering an injection.

(iv) A fluid flowing out of a small hole in a vessel results in a backward thrust on the vessel.

(v) A spinning cricket ball in air does not follow a parabolic trajectory. [NCERT]

Note

According to Bernoulli's theorem, for horizontal flow of fluids,

$$p + \frac{1}{2}\rho v^2 = \text{constant}$$

Therefore, when velocity of a fluid increases, its pressure decreases and *vice-versa*.

Sol. (i) When we blow over a piece of paper, the velocity of air above the paper increases. So, in accordance with Bernoulli's theorem $\left(p + \frac{1}{2}\rho v^2 = \text{constant}\right)$, the pressure of air decreases above the paper. Due to the pressure difference of air between below and above the paper a lifting force acts on paper and hence, it remain horizontal.

(ii) According to equation of continuity, for steady flow of liquid the product of area of cross-section of the tube and velocity of liquid remains constant at each point, i.e. $A_1 v_1 = A_2 v_2$

When we try to close a water tap with our fingers, the area of cross-section of the outlet of water jet is reduced due to the restriction provided by the fingers and therefore, the velocity of water increases and fast jets of water gush through the openings between our fingers.

(iii) According to Bernoulli's theorem,

$$p + \frac{1}{2}\rho v^2 = \text{constant}$$

In this relation, pressure (p) occurs with one power while velocity (v) occurs with two powers. Therefore, the influence of velocity is higher than pressure. The size of the needle controls the velocity of flow and the thumb pressure controls pressure.

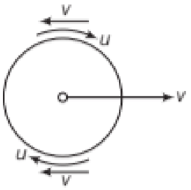
Therefore, size of the needle of a syringe controls flow rate better than thumb pressure.

(iv) A fluid flowing out of a small hole in a vessel have a large velocity and therefore, a large momentum. As no external force is acting, therefore, according to law of conservation of momentum equal momentum in opposite direction and hence, a backward velocity is attained by the vessel. Therefore, a backward thrust $\left(F = \frac{dp}{dt}\right)$ acts on the vessel.

(v) A spinning cricket ball in air does not follow a parabolic trajectory due to Magnus effect.

Let a spinning cricket ball is moving forward with a velocity v and spinning clockwise with velocity u. As ball moves forward, it leaves, a lower pressure region behind it.

To fill this region, air moves backward with velocity v. The layers of air in contact with the ball spin with ball with velocity u. Therefore, the resultant velocity of air above the ball is $(v-u)$ and below the ball is $(v+u)$.



According to Bernoulli's theorem,

$$p + \frac{1}{2}\rho v^2 = \text{constant}$$

Therefore, pressure below the ball becomes lower than above the ball. Due to this pressure difference, a force acts on ball in downward direction. Therefore, the ball follows a curved path in spite of a parabolic trajectory.

30. A non-viscous liquid of constant density $1000\ \text{kgm}^{-3}$ flows in a streamline motion along a tube of variable cross-section. The tube is a kept inclined in the vertical plane as shown in figure. The area of cross-section of the tube at two points P and Q at heights of 2 m and 5 m are respectively, $4\times 10^{-3}\text{m}^2$ and $8\times 10^{-3}\text{m}^2$. The velocity of the liquid at point P is $1\ \text{ms}^{-1}$. Find the work done per unit volume by the pressure and the gravity forces as the fluid flows from point P to Q.

Sol. Given, $\rho = 1000\,\text{kg/m}^3$, $v_1 = 1\ \text{m/s}$, $a_1 = 4\times 10^{-3}\ \text{m}^2$,

$a_2 = 8\times 10^{-3}\ \text{m}^2$, $h_1 = 2\ \text{m}$, $h_2 = 5\ \text{m}$

Apply Bernoulli's theorem,

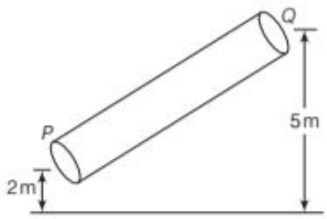
$$p_1 + \frac{1}{2}\rho v_1^2 + g\rho h_1 = p_2 + \frac{1}{2}\rho v_2^2 + g\rho h_2$$

$$(p_1 - p_2) = \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(h_2 - h_1)$$

where,

$(p_1 - p_2)$ = Work done by pressure per unit volume

i.e. $$\left(\frac{W}{\text{Volume}}\right)_p = \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(h_2 - h_1) \quad ...(i)$$



From equation of continuity,

$$a_1 v_1 = a_2 v_2$$

$$v_2 = \frac{a_1 v_1}{a_2} = \frac{4\times 10^{-3}\times 1}{8\times 10^{-3}} = 0.5\ \text{m/s}$$

$$\left(\frac{W}{\text{Volume}}\right)_p = \frac{1}{2}\times 1000\,[0.25 - 1] + 1000\times 10\,(5-2)$$

$$= -375 + 30{,}000 = 29625\ \text{J/m}^3$$

Work done per unit volume by the gravitational force

$$= \rho g(h_1 - h_2)$$

$$= 1000\times 10\,(2-5) = -3\times 10^4\ \text{J/m}^3$$

31. If a sphere of radius r falls under gravity through a liquid of viscosity η, its average acceleration is half that of in starting of the motion. Then, show that the time taken by it to attain the terminal velocity is independent of the liquid density.

Sol. Let the density of sphere's material is ρ and that of liquid is σ. When the sphere just enters in the liquid.

Downward force on the sphere, F = weight of the sphere − weight of the fluid displaced by it.

$$F = \frac{4}{3}\pi r^3\cdot\rho g - \frac{4}{3}\pi r^3\cdot\sigma g$$

$\because$ Mass = Volume × Density $= \frac{4}{3}\pi r^3(\rho - \sigma)g$

$\therefore$ Acceleration of the sphere at this instant,

$$a = \frac{F}{m}$$

$$a = \frac{\frac{4}{3}\pi r^3(\rho-\sigma)g}{\frac{4}{3}\pi r^3\rho} = \left(1 - \frac{\sigma}{\rho}\right)g$$

When the sphere approches to terminal velocity, its acceleration becomes zero.

$\therefore$ Average acceleration of the sphere $= \frac{a+0}{2}$

$$= \frac{\left(1-\frac{\sigma}{\rho}\right)g}{2} = \left(1-\frac{\sigma}{\rho}\right)\frac{g}{2}$$

If time t taken by the sphere to attain the terminal velocity

As we know that,

Terminal velocity, $v = \frac{2}{9}\frac{r^2}{\eta}(\rho - \sigma)g$

$\because$ The sphere falls from rest,

$\therefore$ $u = 0$

Using $v = u + at$

Putting values $\frac{2}{9}\frac{r^2}{\eta}(\rho-\sigma)g = 0 + \left(1-\frac{\sigma}{\rho}\right)\frac{g}{2}\cdot t$

$\therefore$ $$t = \frac{4}{9}\frac{r^2\rho}{\eta}$$

Thus, t is independent of the liquid density.

32. If a liquid is flowing through a horizontal tube, write down the formula for the volume of the liquid flowing per second through it. Water is flowing through a horizontal tube of radius 2r and length 1 m at a rate of 60L/s, when connected to a pressure difference of h cm of water. Another tube of same length but radius r is connected in series with this tube and the combination is connected to the same pressure head. Find out the pressure difference across each tube and the rate of flow of water through the combination.

Sol. The volume of the liquid flowing per second through a horizontal tube, $V = \frac{\pi}{8} \cdot \frac{pr^4}{\eta l}$

where,
r = radius of the tube,
l = length of the tube,
P = pressure difference across the two ends of the tube
and η = coefficient of viscosity of the liquid

$$\because \quad V = \frac{\pi}{8} \cdot \frac{pr^4}{\eta l}$$

In first case,

$$= \frac{\pi}{8} \cdot \frac{h\rho g(2r)^4}{\eta l} \qquad [\because \ p = h\rho g]$$

$$\frac{\pi}{8} \cdot \frac{h\rho g(2r)^4}{\eta l} = 60 \qquad ...(i)$$

In IInd case, the volume of liquid flowing per second V_1, through each tube is equal

$$V_1 = \frac{\pi}{8} \frac{\rho_1(2r)^4}{\eta l} = \frac{\pi \rho_2 (r)^4}{8\eta l} \qquad ...(ii)$$

$$\because \quad \rho_1 + \rho_2 = h\rho g \qquad ...(iii) \text{ [given]}$$

From Eq. (ii),

$$\rho_1 = \frac{\rho_2}{16}$$

Putting this value into Eq. (iii), we get

$$\rho_2 = \frac{16h\rho g}{17}$$

Putting this value of ρ_2 into Eq. (ii)

$$V_1 = \frac{\pi}{8} \cdot \frac{16h\rho g}{17} \cdot \frac{r^4}{\eta l}$$

$$= \frac{1}{17} \cdot \frac{\pi}{8} \cdot \frac{h\rho g}{\eta l}(2r)^4$$

$$= \frac{1}{17} \times 60 \qquad \text{[using Eq. (i)]}$$

$$= 3.53 \text{L}/s$$

33. Glycerine flows steadily through a horizontal tube of length 1.5 m and radius 1.0 cm. If the amount of glycerine flowing per second at one end is 4.0×10^{-3} kg/s. What is the pressure difference between the two ends of the tube? (Density of glycerine = $1.3 \times 10^3 \text{kg/m}^3$ and viscosity of glycerine = 0.83 Pa-s). (You may also like to check, if the assumption of laminar flow in the tube is correct). **[NCERT]**

Sol. Given, length of the tube $(l) = 1.5$ m
Radius of the tube $(r) = 1.0 \text{ cm} = 1 \times 10^{-2}$ cm
Mass of glycerine flowing per second $= 4 \times 10^{-3}$ kg / s
Density of glycerine, $\rho = 1.3 \times 10^3 \text{ kg / m}^3$
Viscosity of glycerine, $\eta = 0.83$ Pa - s
Volume of glycerine flowing per second, $V = \frac{m}{\rho}$

$$\left[\because \text{density} = \frac{\text{Mass}}{\text{Volume}}\right]$$

$$= \frac{4 \times 10^{-3}}{1.3 \times 10^3} \text{m}^3/\text{s} = \frac{4}{1.3} \times 10^{-6} \text{m}^3/\text{s}$$

According to Poiseuille's formula, the rate of flow of liquid through a tube

$$V = \frac{\pi}{8} \frac{pr^4}{\eta l}$$

where, p is the pressure difference between the two ends of the tube.

$$\text{or} \qquad p = \frac{8\eta l V}{\pi r^4} = \frac{8 \times 0.83 \times 1.5 \times 4 \times 10^{-6}}{3.14 \times (1 \times 10^{-2})^4 \times 1.3}$$

$$= 976 \text{ Pa}$$

To check the laminar flow in the tube, the value of Reynold's number should be less than 2000.

Reynold's number, $R_e = \frac{\rho D v_c}{\eta}$

where, v_c is the critical velocity and D is the diameter of the tube.

$$\text{Critical velocity, } v_c = \frac{\text{Volume flowing out per second}}{\text{Area of cross-section}}$$

$$= \frac{m/\rho}{A} = \frac{m}{\rho \pi r^2} \qquad [\because A = \pi r^2]$$

$$\therefore \text{ Reynold's number, } R_e = \frac{\rho D}{\eta} \times \frac{m}{\rho \pi r^2}$$

$$= \frac{2r \times m}{\eta \pi r^2} \qquad [\because D = 2r]$$

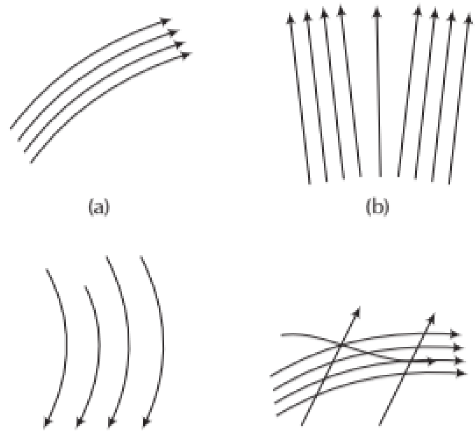
$$= \frac{2m}{\pi r \eta} = \frac{2 \times 4 \times 10^{-3}}{3.14 \times 10^{-2} \times 0.83} = 0.31$$

As $R_e < 2000$, therefore, flow of glycerine is laminar.

ASSESS YOUR TOPICAL UNDERSTANDING

OBJECTIVE Type Questions

1. Which of the following diagrams does not represent a streamline flow? **[NCERT Exemplar]**



(a) (b) (c) (d)

2. In a turbulent flow, the velocity of the liquid molecules in contact with the walls of the tube is

(a) zero (b) maximum
(c) equal to critical velocity (d) may have any value

3. According to Bernoulli's equation,

$$\frac{p}{\rho g} + h + \frac{1}{2}\frac{v^2}{g} = \text{constant}$$

The terms, $\frac{p}{\rho g}$, h and $\frac{1}{2}\frac{v^2}{g}$ are generally called respectively :

(a) Gravitational head, pressure head and velocity head
(b) Gravity, gravitational head and velocity head
(c) Pressure head, gravitational head and velocity head
(d) Gravity, pressure and velocity head

4. A cylinder of height 20 m is completely filled with water. The velocity of efflux of water (in ms^{-1}) through a small hole on the side wall of the cylinder near its bottom is

(a) 10 (b) 20
(c) 25.5 (d) 5

5. We have three beakers A, B and C containing three different liquids. They are stirred vigorously and placed on a table. Then, liquid which is

(a) most viscous comes to rest at the earliest
(b) most viscous comes to rest at the last
(c) most viscous slows down earliest but comes to rest at the last
(d) All of them come to rest at the same time

Answer

1. (d)	2. (d)	3. (c)	4. (b)	5. (a)

Very Short Answer Type Questions

6. What is the effect of temperature on viscosity of a liquid?

7. What is the net weight of a body when it falls with terminal velocity through a viscous fluid?

8. When water flows through a pipe, which layer moves faster?

9. Why the gas bubbles rise up through soda water bottle? Give one reason only.

10. Why the viscosity of the liquid decreases with increase in temperature?

11. Which fundamental law forms the basis of equation of continuity?

SHORT ANSWER Type Questions

12. When does the streamline become turbulent. How?

13. Why does the speed of a whirlwind in a tornado alarmingly high?

14. What do you mean by the term 'terminal velocity' for motion through a viscous medium?

LONG ANSWER Type I Questions

15. The relative velocity between two parallel layers of water is $8\ cms^{-1}$ and the perpendicular distance between them is 0.1 cm. Calculate the velocity gradient. **[Ans.** $80\ cms^{-1}/cm$ **]**

16. Water flows at a speed of $6\ cms^{-1}$ through a pipe of tube of radius 1 cm. The coefficient of viscosity of water at room temperature is 0.01 poise. What is the nature of flow?

[Ans. $R_e = 1200 < 2000$ so, flow is laminar**]**

17. Find the critical velocity for air flowing through a tube of 2 cm diameter. For air, $\rho = 1.3 \times 10^{-3}\ g\ cm^{-3}$ and $\eta = 181 \times 10^{-6}$ poise. **[Ans.** $140\ cms^{-1}$**]**

LONG ANSWER Type II Questions

18. The reading of pressure meter attached with a closed pipe is $3.5 \times 10^5\ Nm^{-2}$. On opening the valve of the pipe, the reading of the pressure meter is reduced to $3.0 \times 10^5\ Nm^{-2}$. Calculate the speed of the water flowing in pipe. **[Ans.** 10 m/s**]**

19. A capillary tube 1 mm in diameter and 20 cm in length is fitted horizontally to a vessel kept full of alcohol. The depth of the centre of capillary tube below the surface of alcohol is 20 cm. If the viscosity and density of alcohol are 0.012 UGS unit and 0.8 gcm^{-3}, respectively, find the amount of the alcohol that will flow out in 5 min. Given that $g = 980\ cms^{-2}$

[**Ans.** 38.4 g]

20. A liquid is kept in cylindrical vessel which is rotated along its axis. The liquid rises at the sides. If the radius of vessel is 0.05 m and the speed of rotation is 2 rev/s, find the difference in height of the liquid at the centre of the vessel and its sides.

[**Ans.** 0.02 m]

21. Water flows through a horizontal pipe whose internal diameter is 2.0 cm, at a speed of 1.0 ms^{-1}. What should be the diameter of the nozzle, if the water is to emerge at a speed of 4.0 ms^{-1}?

[**Ans.** 1.0 cm]

22. Check the dimensional consistancy of the Poiseuille's formula for the laminar flow in a tube

$$Q = \frac{\pi R^4 (p_1 - p_2)}{8\eta l}$$

SUMMARY

- **Thrust** The total force exerted by the liquid at rest, on any surface in contact with it is called the thrust of the liquid on that surface.
- **Pressure**, $p = \frac{\text{Thrust}}{\text{Area}} = \frac{F}{A}$. Its SI unit is N/m^2 or Pascal (Pa).
  Pressure exerted by a liquid column
  $$p = \rho g h$$
  where, h = height of liquid column, g = acceleration due to gravity, ρ = density of liquid.
- **Pascal's law** It states that if an external pressure is applied to an enclosed liquid, it is transmitted undiminished equally to all other points of the liquid and to the walls of container.
- **Atmospheric pressure** The pressure exerted by the atmosphere is called atmospheric pressure.
  At sea-level, we have atmospheric pressure = $1.013 \times 10^5\ Nm^{-2}$
- **Absolute Pressure and gauge pressure** The total or actual pressure p at a point is called absolute pressure. Gauge pressure is the difference between the actual pressure (absolute pressure) at a point and the atmospheric pressure. Thus,
  $$p_g = p - p_a \quad \text{or} \quad p = p_a + p_g$$
  Absolute pressure = Atmospheric pressure + Gauge pressure.
- **Buoyancy** It states that when a body is partially or fully dipped into a fluid, the fluid exerts an upward force on the body called as buoyant force and this phenomenon is called buoyancy.
- **Archimedes' principle** It states that when a body is partially or fully dipped in a fluid at rest, the fluid exerts an upward force of buoyancy equal to the weight of the displaced fluid.
  Weight of the body = Weight of fluid displaced
  $$V_b \rho_b g = V_f \rho_f g$$
  ∴ Denisity of body to fluid, $\frac{\rho_b}{\rho_f} = \frac{V_f}{V_b}$
- **Surface tension** (S) Surface tension is the property of a liquid due to which its free surface behaves like a stretched elastic membrane and tends to have least possible surface area.
  SI unit of surface tension is Nm^{-1} or Jm^{-2}.
- **Capillarity** The phenomenon of rise or fall of a liquid in the capillary tube is called capillarity.
  Height of rise of liquid in capillary is given by $h = \frac{2S\cos\theta}{\rho R g}$ where, R = radius of capillary base.
- **Streamline and Turbulent Flow** Flow of a fluid is said to be streamlined if each element of the fluid passing through a particular point travels along the same path, with exactly the same velocity as that of the preceding element.

A special case of streamline flow is laminar flow, in which a fluid has a steady flow in the form of parallel layer and these do not mix with one another.

- **A turbulent flow** is the one in which the motion of the fluid particles is disordered or irregular. In such a flow, most of the energy used up in maintaining the flow, is spent in causing eddies in the fluid and only a small part of the energy is used for the actual forward flow.
- **Equation of continuity** It states that when an incompressible and non-viscous fluid flows steadily through a tube of non-uniform cross-section, then the product of area of cross-section and the velocity of flow is same at every point in the tube.

$$Av = \text{constant}$$

- **Bernoulli's theorem** It states that the sum of pressure energy, kinetic energy and potential energy per unit volume of an incompressible and non-viscous fluid in a streamlined irrotational flow remains constant at every cross-sectional throughout the liquid flow.

$$p + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$$

where, p = pressure, $\frac{1}{2}\rho v^2$ = kinetic energy per unit volume, ρgh = potential energy per unit volume.

- **Velocity of efflux** (Torricelli's theorem) The speed of liquid coming out through a hole at a depth (h) below the free surface is called velocity of efflux, $v = \sqrt{2hg}$
- **Viscosity** It is the property of a fluid due to which an opposing force comes into play whenever there is relative motion between its different layers.
- **Newton's formula for viscous force** The viscous drag between two parallel layers each of area A and having velocity gradient $\frac{dv}{dx}$ is given by $F = -\eta A \frac{dv}{dx}$

where, η is the coefficient of viscosity of the liquid.

- **Stoke's law** It states that the backward dragging force of viscosity acting on a spherical body of radius r moving with velocity v through a fluid of viscosity η is $F = 6\pi\eta rv$
- **Terminal velocity** It is the maximum constant velocity attained by a spherical body while falling through a viscous medium. The terminal velocity of a spherical body of density ρ and radius r moving through a fluid of density ρ' and viscosity η is given by $v = \frac{2}{9}\frac{r^2}{\eta}(\rho - \rho')g$
- **Reynold's number** It is a dimensionless number which determines the nature of the flow of the liquid. For a liquid of viscosity η, density ρ and flowing through a pipe of diameter D, Reynold's number is given by

$$R_e = \frac{\rho v D}{\eta}$$

If $R_e < 2000$, the flow is streamline or laminar.
If $R_e > 3000$, the flow is turbulent.
If $2000 < R_e < 3000$, the flow is unstable. It may change from laminar to turbulent and *vice-versa*.

- **Poiseuille's formula** The volume of a liquid flowing per second through a horizontal capillary tube of length l, radius r under a pressure difference p across its two ends is given by $V = \frac{\pi}{8}\cdot\frac{pr^4}{\eta l}$, where, η is coefficient of viscosity of the liquid.

CHAPTER PRACTICE

OBJECTIVE Type Questions

1. Pascal's law states that pressure in a fluid at rest is the same at all points, if
(a) they are at the same height
(b) they are along same plane
(c) they are along same line
(d) Both (a) and (b)

2. If two liquids of same masses but densities ρ_1 and ρ_2 respectively are mixed, then density of mixture is given by
(a) $\rho = \frac{\rho_1 + \rho_2}{2}$
(b) $\rho = \frac{\rho_1 + \rho_2}{2\rho_1\rho_2}$
(c) $\rho = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2}$
(d) $\rho = \frac{\rho_1\rho_2}{\rho_1 + \rho_2}$

3. The excess pressure inside an air bubble of radius r just below the surface of water is p_1. The excess pressure inside a drop of the same radius just outside the surface is p_2. If T is surface tension, then
(a) $p_1 = 2p_2$
(b) $p_1 = p_2$
(c) $p_2 = 2p_1$
(d) $p_2 = 0, p_1 \neq 0$

4. In a soap bubble, pressure difference is
(a) $\frac{2S_{la}}{r}$ (b) $\frac{4S_{la}}{r}$ (c) $\frac{S_{la}}{r}$ (d) $\frac{8S_{la}}{r}$

5. Along a streamline, **[NCERT Exemplar]**
(a) the velocity of a fluid particle remains constant
(b) the velocity of all fluid particles crossing a given position is constant
(c) the velocity of all fluid particles at a given instant is constant
(d) the speed of a fluid particle remains constant

6. An ideal fluid flows through a pipe of circular cross-section made of two sections with diameters 2.5 cm and 3.75 cm. The ratio of the velocities in the two pipes is **[NCERT Exemplar]**
(a) 9 : 4
(b) 3 : 2
(c) $\sqrt{3} : \sqrt{2}$
(d) $\sqrt{2} : \sqrt{3}$

7. As the temperature of water increases, its viscosity
(a) remains unchanged
(b) decreases
(c) increases
(d) increases or decreases depending on the external pressure

8. Reynold's number (R_e) can be defined as
(a) $\frac{\rho\eta}{vd}$
(b) vd/ρ
(c) $\frac{\rho vd}{\eta d}$
(d) $d\rho v/\eta$

ASSERTION AND REASON

Direction (Q.Nos. 9-18) *In the following questions, two statements are given- one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below*
(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) Assertion is true but Reason is false.
(d) Assertion is false but Reason is true.

9. **Assertion** In steady flow, the velocity of each passing fluid particle remains constant in time.

Reason Each particle follows a smooth path and the paths of the particle do not cross each other.

10. **Assertion** In streamline flow, $A \times v$ is constant.

Reason For incompressive flow, mass in = mass out.

11. **Assertion** The stream of water flowing at high speed from a garden horse pipe tend to spread like a fountain when held vertically up, but tends to narrow down when held vertically down.

Reason In any steady flow of an incompressible fluid, the volume flow rate of the fluid remains constant.

12. **Assertion** The shape of an automobile is so designed that its front resembles the streamline pattern of the fluid through which it moves.

Reason The resistance offered by the fluid is proportional to area.

13. **Assertion** The machine parts are jammed in winter.

Reason The viscosity of the lubricants used in the machine increases at low temperature.

14. **Assertion** For Reynold's number $R_e > 2000$, the flow of fluid is turbulent.

Reason Inertial forces are dominant as compared to the viscous forces.

15. **Assertion** A fluid will stick to a solid surface.

Reason Surface energy between fluid and solid is smaller than the sum of surface energies.

16. **Assertion** Smaller drop of liquid resists deforming forces better than the larger drops.

Reason Excess pressure inside a drop is directly proportional to its surface area.

17. **Assertion** The surface of water in the capillary tube is concave.

Reason The pressure difference between two sides of the tube is $\frac{2S}{a}\cos\theta$.

18. **Assertion** Ploughing a field reduces evaporation of water from the ground beneath.

Reason Results in lowering of surface area open to sunlight.

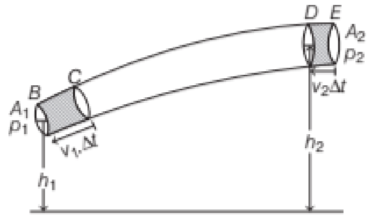
CASE BASED QUESTIONS

Direction (Q. Nos. 19-20) *These questions are case study based questions. Attempt any 4 sub-parts from each question.*

19. **Fluid Dynamics**

Consider the flow at two regions 1 (i.e. BC) and 2 (i.e. DE). Consider the fluid initially lying between B and D. In an infinitesimally time interval Δt, this fluid would have moved. Suppose v_1 is the speed at B and v_2 at D, then fluid initially at B has moved a distance $v_1\Delta t$ to C ($v_1\Delta t$ is small enough to assume constant cross-section along BC).

In the same interval Δt, the fluid initially at D moves to E, a distance equal to $v_2\Delta t$. Pressures p_1 and p_2 act as shown on the plane faces of areas A_1 and A_2 binding the two regions as shown in figure



(i) The work done on the fluid at BC is

(a) $p_1 A_1 \Delta t$ (b) $p_1 v_1 \Delta t$
(c) $p_1 \Delta V$ (d) $A_1 \Delta V$

(ii) The work done on the fluid of DE is

(a) $p_2 \Delta V$ (b) $-p_2 \Delta V$
(c) $p_1 A_2 \Delta t$ (d) $p_2 v_2 \Delta t$

(iii) Total work done on the fluid is

(a) $\frac{p_1 - p_2}{2}\Delta V$ (b) $p_2 \Delta V$
(c) $p_1 \Delta V$ (d) $(p_1 - p_2)\Delta V$

(iv) The change in its kinetic energy is

(a) $\frac{1}{2}\Delta V(v_2^2 - v_1^2)$

(b) $\frac{1}{2}\rho\Delta V(v_2^2 - v_1^2)$

(c) $\rho\Delta V(v_2^2 - v_1^2)$

(d) $\frac{1}{2}\rho\Delta V(v_2^2 + v_1^2)$

(v) Expression of Bernoulli's equation is

(a) $p + \frac{1}{2}\rho v^2 = \text{constant}$

(b) $p + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$

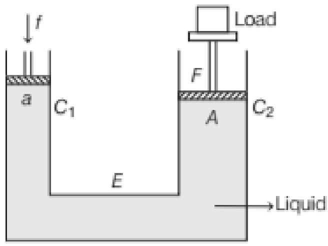
(c) $p + \rho v^2 + \frac{1}{2}\rho g h = \text{constant}$

(d) $\frac{1}{2}\rho v^2 + \rho g h = \text{constant}$

20. **Hydraulic Lift**

Hydraulic lift is an application of Pascal's law. It is used to lift heavy loads. It is a force multiplier.

So, when small forces applied on the smaller piston (acting downward) will be appearing as a very large force (acting upward) on the larger piston. As a result of it, a heavy load placed on the larger piston is easily lifted upwards.



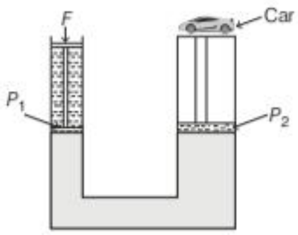
(i) Pascal's law states that pressure in a fluid at rest is the same at all points, if
(a) they are at the same height
(b) they are along same plane
(c) they are along same line
(d) Both (a) and (b)

(ii) Pressure is applied to an enclosed fluid as shown in the above figure. It is
(a) increased and applied to every part of the fluid
(b) diminished and transmitted to the walls of the container
(c) increased in proportion to the mass of the fluid and then transmitted
(d) transmitted unchanged to every portion of the fluid and the walls of container

(iii) Pressure at a point inside a liquid does not depend on
(a) the depth of the point below the surface of the liquid
(b) the nature of the liquid
(c) the acceleration due to gravity at that point
(d) total weight of fluid in the beaker

(iv) A hydraulic lift has 2 limbs of areas A and $2A$. Force F is applied over limb of area A to lift a heavy car.

If distance moved by piston P_1 is x, then distance moved by piston P_2 is



(a) x (b) $2x$ (c) $\frac{x}{2}$ (d) $4x$

(v) If work done by piston in the given figure on fluid is W_1, then work done by fluid in limbs on piston P_2 is
(a) $\frac{W_1}{4}$ (b) $4W_1$ (c) $\frac{W_1}{2}$ (d) W_1

Answer

1. (a)	2. (c)	3. (b)	4. (b)	5. (b)
6. (a)	7. (b)	8. (d)	9. ()	10. ()
11. (a)	12. (a)	13. (a)	14. (a)	15. (a)
16. (c)	17. (a)	18. (c)		

19. (i) (c) (ii) (b) (iii) (d) (iv) (b) (v) (b)
20. (i) (a) (ii) (d) (iii) (d) (iv) (c) (v) (d)

VERY SHORT ANSWER Type Questions

21. How is the surface tension of a liquid explained on the basis of intermolecular forces?

22. From where the energy comes when a liquid rises against gravity in a capillary tube?

SHORT ANSWER Type Questions

23. A thin wire is bent in the form of a ring of diameter 3.0 cm. The ring is placed horizontally on the surface of soap solution and then raised up slowly. How much upward force is necessary to break the vertical film formed between the ring and the solution? $T = 3.0 \times 10^{-2} \text{Nm}^{-1}$
[**Ans.** 5.66×10^{-3} N]

24. Prove that there is always excess of pressure on the concave side of the meniscus of a liquid. Obtain expression for the excess of pressure inside a liquid drop.

LONG ANSWER Type I Questions

25. Two cylindrical vessels placed on a horizontal table contain water and mercury, respectively up to the same heights. There is a small hole in the walls of each of the vessels at half the height of liquids in the vessels. Find out the ratio of the velocities of efflux of water and mercury from the holes. Which of the two jets of liquid will fall at a greater distance on the table from the vessel? Relative density of mercury with respect to water = 13.6

26. Water enters a horizontal pipe of non-uniform cross-section with a velocity or 0.6 ms^{-1} and leaves the other end with a velocity of 0.4 ms^{-1}. At the first end, pressure of water is 1200 Nm^{-2}. Calculate the pressure of water at the other end. Density or water is 1000 kgm^{-3}. [**Ans.** 1300 N/m^2]

27. 27 identical drops of water are falling down vertically in air each with a terminal velocity 0.15 ms^{-1}. If they combine to form a single bigger drop, what will be its terminal velocity?
[**Ans.** 1.35 m/s]

28. Two soap bubbles have radii in the ratio 2 : 3. Compare the excess of pressure inside these bubbles. [**Ans.** 4/9]

29. If the sap in tree behaves like water in glass capillaries, what must be the diameter of the pores in the trunk of a teak tree 30 m high for the sap to reach the tap. Surface tension of sap $= 72 \times 10^{-3}$ N/m and angle.

LONG ANSWER Type II Questions

30. If a number of little droplets of water, all of the same radius r, coalesce to form a single drop of radius R, show that the rise in temperature will be given by $\frac{3T}{\rho J}\left(\frac{1}{r} - \frac{1}{R}\right)$, where T is the surface tension of water and J is the mechanical equivalent of heat.

31. If the terminal speed of a sphere of gold (density $= 19.5\,\text{kg/m}^3$) is 0.2 m/s in viscous liquid (density $= 19.5\,\text{kg/m}^3$) is 0.2 m/s in viscous liquid (density $= 1.5\,\text{kg/m}^3$).

Find out the terminal speed of a sphere of silver (density $= 10.5\,\text{kg/m}^3$) of the same size in the same liquid.

[**Ans.** 1.76×10^{-2} N/m]

32. During blood transfusion of a patient, the bottle of the blood is adjusted on a stand in such a way that blood is 1.3 m above the needle which has a radius of 0.18 mm and length of 3 cm. Calculate the viscosity of the blood if 4.5 cc of blood passes through needle in 60 s. Given density of blood is $1.02\,\text{g cm}^{-3}$. [**Ans.** 0.0238 Poise]